Compound Propositions: Hypothetical Propositions

I. Lesson

To conclude our study of compound propositions, we'll be looking at the *hypothetical proposition* in this lesson. In addition to this, we'll also take a look at what happens when we combine some of the compound propositions we've been discussing.

- **Hypothetical Proposition**: a proposition consisting of two other propositions, linked together in an if-then structure. The form of this proposition is "if *p*, then *q*."

In the variable notation above, we call *p* the *antecedent* and *q* the *consequent*. The last of the compound propositions, the hypothetical, links two propositions by asserting that the truth of the *antecedent* implies the truth of the *consequent*. It is *hypothetical* because it states that if, *hypothetically*, the *antecedent* is true, then it is the case that the *consequent* is true. Only the *antecedent* is hypothetical. These propositions will take the form "If *p*, then *q*" in variable notation. For a simple example, consider: "If frogs are amphibians, then they are animals". If it is true that frogs are amphibians (*antecedent*), then it is true that they are animals (*consequent*).

Truth Conditions for Hypothetical Propositions: The truth of these propositions is a bit more complicated than the prior two compound forms, as they will only be false in situations where the *antecedent* is true and the *consequent* turns out false. For example, the hypothetical proposition "If I am a human, then I am an animal" is false *only when* "I am a human" is true but "I am an animal" is false. This is because the hypothetical seeks to assert that the *antecedent* cannot be true without the *consequent* being true. Because of this, the only situation in which the hypothetical is false will be exactly when the *antecedent* is true, but the *consequent* still turns out to be false.

p	*q*	"If *p*, then *q*"
True	True	True
False	True	True
True	False	False
False	False	True

As the table shows, the only situation when the hypothetical proposition, "if *p* then *q*," is false is when *p* is true and *q* is false. Strangely enough, this means that whenever *p* is false, "if *p* then *q*" is true no matter whether *q* is true or false! For example, take *p* to be "Obama is president" and *q* to be "unicorns are real," then the statement "if *p* then *q*" is "if Obama is president, then unicorns are real." Since *p* is false—Obama is no longer president—then the statement as a whole is true under these logical conditions, even though, as far as we know, unicorns aren't real. Likewise, "if *p* then *q*" is always true when *q* is true.

II. Examples

i. Hypothetical Compound Propositions

Now, let's consider a hypothetical compound proposition: "If frogs are amphibians, then they are animals".

The hypothetical is true because amphibians are a type of animal, so a frog cannot be an amphibian without being an animal. The truth of the *antecedent* implies the truth of the *consequent*.

Now let's consider the alternative example "If the animal is an amphibian, then it is a frog".

This hypothetical is false, as there are amphibians that are not frogs, so the *antecedent* may be true without the *consequent* having to be true.

Finally, let's take the case "If scorpions are insects, then frogs are amphibians".

In this case the hypothetical is true despite the fact that the *antecedent* is false (Scorpions are arachnids not insects). The truth of this proposition lies in the fact that frogs are in fact amphibians, so the hypothetical proposition would be true if scorpions turned out to be insects or not. Specifically, if scorpions were in actuality insects, then this hypothetical proposition could not be true without the proposition "Frogs are amphibians" being true, and we know the latter to be true.

III. Exercises

Exercise 1

Directions: Give the variable notation of the hypothetical proposition. Explain, in your own words, what variable notation is, and how it applies to the hypothetical proposition.

Exercise 2

Directions: fill out the following tables describing whether the hypothetical, disjunctive, and conjunctive complex propositions are true. Try to do this from memory, not looking at tables from earlier in this lesson or from earlier lessons.

1.

p	***q***	**"If *p*, then *q*"**
True	True	
False	True	
True	False	
False	False	

2.

p	***q***	**"*p* or *q*" (exclusive or)**
True	True	
False	True	
True	False	
False	False	

3.

p	***q***	**"*p* or *q*" (inclusive or)**
True	True	
False	True	
True	False	

False	False	

4.

p	***q***	**"*p* and *q*"**
True	True	
False	True	
True	False	
False	False	

Exercise 3

Directions: Give three examples of true hypothetical propositions.

Ex. If pigs are mammals, then they are animals.

1.)
2.)
3.)

Exercise 4

Directions: Determine whether the following compound propositions are true or false, and justify your answer.

Ex. If birds have wings, then pigs have wings.

Answer: False, as only the *antecedent* is true. However, the rules of the hypothetical proposition mandate that the truth of this proposition requires the *consequent* to be true if the *antecedent* is.

1.) If puppies are cute, then grown dogs are cute.
2.) If the sky is clear, then it is not raining. (Assume the truth of the *consequent*)

3.) If balloons float, then the gas inside them is heavier than air.
4.) If sharks are mammals, then they can swim.
5.) If Aristotle was the first formal logician, then there are no formal logicians born before him.

Extra Challenge: Complex Compound Propositions

Finally, it should be understood that, as compound propositions are formed through the combination of two simpler propositions, they themselves can be used to form more complex compound propositions. Some examples from natural language could be: "If it is raining or snowing, then my car is covered in water" and "I have a dog and I have a cat, or I have a hamster." The first sentence is a disjunctive proposition combined with a hypothetical proposition, and the second is a conjunctive proposition combined with a disjunctive proposition. While this will not be rigorously tested in this logic course, we can also think of these complex propositions in terms of more complex tables:

p	***q***	***r***	***p* or *q***	**(*p* or *q*) and *r***
True	True	True	True	True
True	True	False	True	False
True	False	True	True	True
True	False	False	True	False
False	True	True	True	True
False	True	False	True	False
False	False	True	False	False
False	False	False	False	False

How ought we read this? This table is helpful, because one only needs to look at two columns at a time in order to fill it out. In order to fill out the blue column below, we only need to look at the green columns. That is, in order to evaluate whether '*p* or *q*' is true, we only need to look at whether *p* is true and whether *q* is true.

p	***q***	***r***	***p* or *q***	**(*p* or *q*) and *r***

True	True	True	True	True
True	True	False	True	False
True	False	True	True	True
True	False	False	True	False
False	True	True	True	True
False	True	False	True	False
False	False	True	False	False
False	False	False	False	False

Finally, in order to fill out the final column on the right, we only need to look at the green columns below. That is, if we wish to figure out if '*p* or *q*, and *r*' is true, then we simply need to look at whether '*p* or *q*' is true and whether *r* is true

p	***q***	***r***	***p* or *q***	**(*p* or *q*) and *r***
True	True	True	True	True
True	True	False	True	False
True	False	True	True	True
True	False	False	True	False
False	True	True	True	True
False	True	False	True	False
False	False	True	False	False
False	False	False	False	False

We can make compound propositions infinitely large, and therefore we can (theoretically) make tables

infinitely large. However, this course in logic will not deal with such large complex propositions.

Concrete Examples

Some examples from natural language of these types of propositions could be: "If it is raining or snowing, then my car is covered in water" and "I have a dog and a cat, or I have a hamster". Within both of these, as is commonly done in natural language, compound propositions are abbreviated into what appears to be a single simple proposition. From our examples above, "It is raining or snowing" really represents the disjunctive proposition "It is raining or it is snowing".

In order to understand the conditions for the truth of these types of statements, one must go to the simplest compound proposition, from which the complex proposition is composed, and determine its truth or falsity. Once this has been done, one should continue onto the next simplest compound proposition and determine the same; in continuing this process of stepping up to the next level of complexity eventually the truth or falsity of the whole proposition should be revealed. This process was exemplified above. From our second example in this section, "I have a dog and a cat, or I have a hamster", let's assume I have only a dog, then the compound proposition "I have a dog and I have a cat" will be false based on the truth conditions for conjunction discussed above. Next, let's assume I do have a hamster, then the proposition "I have a dog and a cat, or I have a hamster" becomes a simple disjunction with one of its disjuncts being false, and the other being true. As we know from our discussion of disjunction, this will mean this complex compound proposition is true. (We take the disjunction to be inclusive here).

Exercise 5 (Challenge Question)

Directions: Determine whether the following complex compound propositions are true or false. Ex.

If you have ten toes, then five are on the left foot and five are on the right.

Answer: Standardly, this proposition can be assumed to be true, as usually, having ten toes implies that five are on each foot.

1.) Apples are fruit or they are not, and fruits have seeds.

2.) If a person can swim or float, then they will drown in water.

3.) Water is composed of Hydrogen and Oxygen, or It is composed of Helium and Aluminum.

Exercise 6 (Challenge Question)

Directions: Fill out the following table regarding whether or not "if *p* then *q*, or *r*" is true. Think of the 'or' here as an 'exclusive or'

p	***q***	***r***	**"If *p*, then *q*"**	**(if *p*, then *q*) or *r***
True	True	True		
True	True	False		
True	False	True		
True	False	False		
False	True	True		
False	True	False		
False	False	True		
False	False	False		

IV. Conclusion

In this lesson, we discussed the hypothetical proposition, which claims to assert that the truth of a proposition (*consequent*) is implied by the truth of a preceding proposition (*antecedent*). We also discussed the possibility of combining multiple compound propositions to form complex compound propositions. The truth of these propositions can be found using the same processes studied in the preceding lessons, starting at the simplest propositions and building up.

V. Learning Goals: Following this lesson, the students should be able to:

1.) identify the form of the hypothetical proposition.
2.) construct hypothetical and complex compound propositions out of other propositions.
3.) state the conditions for the truth the hypothetical proposition as well as any form of complex compound proposition

Answer Key:

Exercise 1

Hypothetical: "If *p*, then *q*"

Exercise 2

1.

p	***q***	**"If *p*, then *q*"**
True	True	True
False	True	True
True	False	False
False	False	True

2.

p	***q***	**"*p* or *q*" (exclusive or)**
True	True	False
False	True	True
True	False	True
False	False	False

3.

p	***q***	**"*p* or *q*" (inclusive or)**
True	True	True
False	True	True
True	False	True
False	False	False

4.

p	***q***	**"*p* and *q*"**
True	True	True
False	True	False
True	False	False
False	False	False

Exercise 3

Answers will vary widely, however the compound propositions must follow the basic form guidelines for the kind of proposition discussed above: "If *p*, then *q*".

Exercise 4

1.) This is true, as both the antecedent and the consequent are true.
2.) Given the truth of both the antecedent and the consequent, this proposition will be true.
3.) This is false, as the antecedent is true, though the consequent is false.
4.) This proposition is of the more confusing form of the hypothetical, since because the consequent is true, regardless of the antecedent, this proposition must be true.
5.) This proposition is true, as Aristotle being the first formal logician implies no formal logician could be born before him.

Exercise 5 (Challenge Question)

1.) This is true as the simple disjunction "Apples are fruit or they are not" has one true disjunct and one false one, so it is true. This will make the conjunction as a whole true, as the other conjunct "all fruits have seeds" is true, so the conjunction has two true conjuncts.
2.) This proposition is false. This can be seen by assuming the truth of the disjunction "a person can swim or float"; once this is assumed, the student should see that being able to swim and being able to float both implies, on their own, that said swimmer or floater will not drown.
3.) This proposition is true as it is a disjunction with one true disjunct and one false one. The first disjunct is a true conjunction, as water is composed of Hydrogen and it is composed of oxygen. The second is a false conjunction, as water is not composed of Helium and it is not composed of Aluminum.

Exercise 6 (Challenge Question)

p	***q***	***r***	**"If *p*, then *q*"**	**(if *p*, then *q*) or *r***
True	True	True	True	False
True	True	False	True	True
True	False	True	False	True
True	False	False	False	False
False	True	True	True	False
False	True	False	True	True
False	False	True	True	False
False	False	False	True	True