The Three Building Blocks of Aristotelian Logic - Propositions

"We must determine what a premise is, what a term is, and what sort of deduction is complete and what sort incomplete.'"

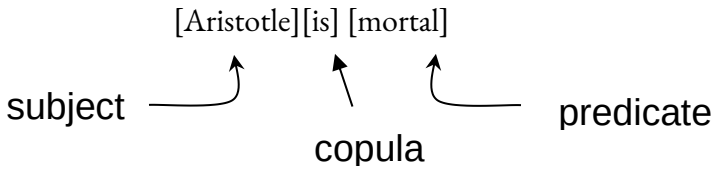
(Aristotle, *Prior Analytics*, transl. Robin Smith, 2012)

I. Lesson

A helpful way to think about propositions is as follows:

- **Propositions:** a *proposition* is whatever a *declarative* sentence expresses, and it can either be true or false.
- **What is the Structure of a Proposition?**

From the perspective of Aristotelian logic, propositions take a specific form: first a *subject*, followed by a *copula,* which introduces the *predicate*.



1. Propositions can be True or False

Unlike terms, propositions can either be **true** or **false**, and are *always* one of the two.

We call **true** and **false** *truth-values*. A proposition *must* be either **true** or **false**. That is, it cannot be both true and false, and it cannot be neither true nor false.

2. How do we figure out if a proposition is true or false?

In Aristotelian logic, we say that a proposition is true if and only if it corresponds to the facts of the matter that it describes. For example, "Aristotle is mortal" is true if and only if it is actually the case that Aristotle is mortal - if we go out in the world, "check" if Aristotle is mortal, and discover that he actually is mortal, then "Aristotle is mortal" is true. When we do this "checking", we say that we

evaluate the proposition. This notion of "evaluation" picks up on the notion of truth-*values*. True and false are here taken to be "values."

Sometimes, instead of checking, we're instead interested to see what we can deduce from a proposition *if* it is true or *if* it is false. We suppose that it's true (or false) and see where that leads us. Put differently, sometimes we don't want to *evaluate* a proposition directly, but instead we want to hypothetically give it a truth-value and see what we can learn. In such cases, we say that we *assign* a truth-value to a proposition. We can think, "What follows if 'America lost the Revolutionary War' is true?" or "What follows if 'all men are mortal' is false?"

[Activity] What would it take for "Aristotle is mortal" to be false?

[Activity] Think about the connection between the two truth-values. If we know that "Aristotle is mortal" is true, is there another proposition that we can subsequently deduce is false?

3. Negation

The connection hinted at in the last activity turns out to be really important in logic. It has to do with *negation*. The negation of a proposition is what we get when we take a proposition and say that whatever it expresses is *not* the case. For example, the negation of "Aristotle is mortal" is "It's not the case that Aristotle is mortal." In this case, we can achieve the same result by *denying the predicate of the subject*. In other words, the negation of "Aristotle is mortal" is "Aristotle is *not* mortal." It is worth noting that this method of simply *denying the predicate* does not always yield the negation of a proposition. We will deal with this in *Section III: Aristotle's Logic of Propositions*.

Looking at our initial definition, we can see that the negation of a proposition is also a proposition itself: it's either true, or false. With this in mind, it's easy to see the connection between a proposition and its negation: when the proposition has one truth-value, its negation has the other one. Specifically, since "Aristotle is mortal" is true, "It's not the case that Aristotle is mortal" is false. *Very importantly*, the "double-negation" of a proposition yields the same proposition. For instance, "It's not the case that it's not the case that Aristotle is mortal" is really just saying "Aristotle is mortal." This result is very important in logic.

Examples

As we have seen, propositions can *affirm* or *deny* predicates of subjects. It's important to keep in mind that affirming a predicate of a subject doesn't get you a true proposition in all cases, just as denying a predicate of a subject doesn't get you a false proposition in all cases. Consider the next example:

1. Aristotle is female.

This proposition would be true, according to our earlier standard, if and only if it actually is the case that Aristotle is female. However, Aristotle is male. Therefore, "Aristotle is female" is *false* and its negation, "Aristotle is not female" is *true*. It should be clear by now that negation is a kind of switch between the two truth-values: take a true proposition and negate it - you'll get a false proposition; take a false proposition and negate it - you'll get a true proposition. Furthermore, we can also see now that whenever we take a proposition and its negation, if the first is true, the second *has to be* false; and if the first is false, the second *has to be* true. We'll have more to say about this result at the very end of our course.

At this point, you might be asking yourself about what would happen if we took a proposition in which a predicate is denied of a subject and then negated *that*. Look at the following:

2. Aristotle is not female.

According to our earlier explanation, the negation of example 4 would look something like this: "Aristotle is not not female" or "It's not the case that Aristotle is not female." These look quite strange, but luckily, we can simplify them to a more familiar expression. Take the former. We know that "Aristotle is not female" is true and we also know that negation switches truth-values. Therefore, we know that "Aristotle is not not female" - the negation of "Aristotle is female" - is false. But we know that "Aristotle is female" is also false from example 3, that its negation is *exactly* "Aristotle is not female,". So, when we say "Aristotle is not not female," we're just flipping back to "Aristotle is female" in a roundabout way.

The above is an instance of the kind of reasoning we sometimes do in logic: we have some information and we leverage it to get other, hopefully clearer information. What we learned from example 4 is simple to state: two negations get you an affirmation. In logicians' talk, this is called *the law of double negation*.

II. Exercises

1. State the definition of a proposition in your own words.
2. What makes a proposition true? What makes a proposition false?
3. Connect the propositions in the first column to those in the second column that say the same thing.

i. Beth is young.	a. Beth is a person who noticeably exceeds average height.
ii. Beth is tall.	b. Beth is a person whose presence is enjoyable.
iii. Beth is fun.	c. Beth is in a predicament which demands physical rest.
iv. Beth is sleepy.	d. Beth is not a person who has lived many years.

4. Evaluate the truth-values of the following propositions, negate them, and simplify where appropriate:
    a. It's not the case that it's not the case that philosophy is an academic subject.
    b. The sky is green.
    c. Washington D.C. is the capital of the United States.
    d. It's not the case that Europe is a continent.
5. In your own words, state the law of double negation.

III. Conclusion

Propositions are the things we express when we use declarative sentences. To build such assertions, we need to use terms in a particular way: a subject term, a predicate term, and a copula to put them together. Constructing propositions out of terms gets us a kind of statement that has a truth-value. Evaluating truth-values by checking against reality and using negation to switch truth-values tells us interesting things about propositions and, in consequence, about the world. But propositions taken in isolation can only get us so far; we want to see what happens when we put propositions together, how they connect, and what further things we can learn by investigating connections between them. This will get us *arguments*. In the next lesson, we'll take a look at a specific form of argument we can get by putting propositions together, namely *the syllogism*. This will turn out to be very important for the rest of our course.

IV. Lesson Goals

At the end of this lesson, students will be able to:

- build propositions out of terms;
- evaluate the truth-values of propositions;
- negate propositions internally;
- grasp pre-theoretic versions of the principle of bivalence and the principle of excluded middle;
- use double negation;

Answer Key:

Exercise 1

Answers may vary. Our suggested definition is as follows: a *proposition* is whatever a declarative sentence expresses, and it can be either true, or false.

Exercise 2

A proposition is true if and only if its content describes an actual state of affairs in the world. A proposition is false if and only if its content does not describe an actual state of affairs in the world.

Exercise 3

i.-d.; ii.-a.; iii.-b.; iv.-c.

Exercise 4

a. is true; simplifies to "Philosophy is an academic subject." its negation: "Philosophy is not an academic subject." or "It's not the case that philosophy is an academic subject."

b. is false; its negation: "The sky is not green." or "It's not the case that the sky is green."

c. is true; its negation: "Washington D.C. is not the capital of the United States," or "It's not the case that Washington D.C. is the capital of the United States."

d. is false; its negation: "It's not the case that it's not the case that Europe is a continent." or "It's not the case that Europe is not a continent." Both negations simplify to "Europe is a continent."

Exercise 5

Answers may vary. Our suggested statement of double negation in the text is as follows: two negations result in an affirmation.