Compound Propositions: Disjunctive Propositions

I. Lesson

Continuing from our study of *conjunctive* propositions last time, within this lesson, we'll be looking at *disjunctive* propositions.

- **Disjunctive Proposition:** a proposition consisting of at least two other, categorical propositions, linked together by "or." The simplest form of this proposition is "*p* or *q*."

Disjunctive Compound Propositions:
In variable notation, propositions of this form will appear as "*p* or *q*", with "*p*" and "*q*" both being separate propositions. For example, let's take the propositions:

(1) "A frog is an amphibian"
and
(2) "A frog is a reptile."

We can form the disjunctive proposition (1 or 2): "A frog is an amphibian *or* a frog is a reptile". We call the propositions (1) and (2) *disjuncts* of the larger *disjunctive proposition*.

Truth Conditions for Disjunctive Propositions: As is the case in natural language, the "or" propositions require that at least one of the *disjuncts* is true. But "or" has some ambiguity in natural language, and this causes a bit of confusion in the interpretation of disjunctive propositions. As with our use of "or" in common speech, disjunctive propositions can be interpreted *inclusively* or *exclusively*.

Intuitively speaking, "or" is *inclusive* if both disjuncts can be true together. Suppose your friend has been making dinner and there's ice cream for dessert. She says "you can have chocolate ice cream or vanilla." In many contexts, this means that you can have either chocolate or vanilla *or both*! Perhaps you like both ice cream flavors so much that you would rather have a mix than have one of them. The exclusive "or" doesn't permit this. Again, here's an intuitive example. Suppose your friend is looking for her book. You say "well, either you can take the northern route home or you can take the southern route home." Here the assumption is that these two scenarios cannot be true together. Either your friend will take

the northern road home or the summer road home, but not both.

'Inclusive or': In its *inclusive* interpretation, a disjunctive proposition will be true when either *disjunct* is true alone, or when both *disjuncts* are true together. We can look at this, like we did with conjunctive propositions, in a table:

p	***q***	**"*p* or *q*"**
True	True	True
False	True	True
True	False	True
False	False	False

Recall that to read this table, starting with the *first row with truth-values*, we say: "if *p* is true and *q* is true, then '*p* or *q*' is true." Next, starting now with the *next* row, we say: "if *p* is false and *q* is true, then '*p* or *q*' is true." As we can see, the *only time* that "*p* or *q*" is false is if both of the *disjuncts*, *p* and *q*, are false.

'Exclusive or': In its *exclusive* interpretation, a disjunctive proposition will only be true when either one *disjunct* is true *or* the other is true, but not when both are. Under either interpretation, the disjunction will be false when both *disjuncts* are false. This could be formalized in variable notation as "*p* or* *q*". In other words, "*p* or *q*, *and not both p* and *q*." We can also interpret 'exclusive or' as saying "one and only one of either *p* or *q* is true."

p	***q***	**"*p* or* *q*"**
True	True	False
False	True	True
True	False	True
False	False	False

Here we can see that the 'exclusive or' table looks identical to the 'inclusive or' table, except for the highlighted row. This makes sense, because the highlighted row is the instance in which both *p* and *q* are true. Remember, we can think of the 'exclusive or' as "*p* or *q*, *and not both p* and *q*."

II. Examples

Now let's consider a disjunctive proposition. Firstly, let's look at "A frog is an amphibian or a frog is a reptile".

This happens to be true, whether we use an 'inclusive or' or an 'exclusive or'. This is because only the first of the two *disjuncts* is true, and when only one of the *disjuncts* is true, both the 'inclusive' and 'exclusive' or statements turn out true.

Consider alternatively the proposition "A frog is an amphibian or a crocodile is a reptile".

This proposition is true under the inclusive interpretation, as both of its *disjuncts* are true. However, under the exclusive interpretation, this proposition is false, as the exclusive disjunctive compound proposition requires that either the first or the second of its *disjuncts* are true but not both. In contrast to this, the proposition "A frog is a reptile or a crocodile is an amphibian" is false according to either account of disjunction.

III. Exercises

Exercise 1

Directions: Give the variable notation of the disjunctive proposition. Explain, in your own words, what variable notation is, and how it applies to the disjunctive proposition.

Exercise 2

Directions: Give three examples of true disjunctive propositions. State whether each uses the exclusive or inclusive "or." Say what it means to take that interpretation of "or."

Ex. "Pigs can fly or oaks are trees." This example uses an exclusive "or." In other words, it is either the case that (1) pigs can fly or (2) oaks are trees, but not both (1) and (2). (Note that the student can interpret this with an inclusive "or" as well.)

1.)

2.)

3.)

Exercise 3

Directions: fill out the following tables describing whether the disjunctive and conjunctive complex propositions are true. Try to do this from memory, not looking at tables from earlier in this lesson or from earlier lessons. Remember, "*p* or* *q*" represents an exclusive "or," and "*p* or *q*" represents an inclusive "or."

1.

p	***q***	**"*p* or* *q*"**
True	True	
False	True	
True	False	
False	False	

2.

p	***q***	**"*p* or *q*"**
True	True	
False	True	
True	False	
False	False	

3.

p	***q***	**"*p* and *q*"**
True	True	
False	True	
True	False	

False	False	

Exercise 4

Directions: Determine whether the following compound propositions are true or false, and justify your answer. Also, specify if you are interpreting the sentence to be an inclusive or exclusive disjunction where needed.

Ex. Birds have wings or Pigs have wings.

Answer: True, as only the first of the two *disjuncts* is true, however the rules of disjunction state that only one of the two *disjuncts* must be true to render the whole proposition true.

1.) Puppies are cute or kittens are cute.
2.) The Atlantic Ocean is green or Canada is in South America.
3.) Balloons can float or humans are primates.
4.) Sharks cannot smell blood or humans have five left feet.
5.) Aristotle was the first formal logician or Aristotle was not the first formal logician.

IV. Conclusion

Within Aristotelian logic, compound propositions are incredibly useful, as often we wish to assert more than one claim in the same proposition. In this lesson, we discussed the disjunctive proposition, which can be divided into two types. The first asserts the truth of one and only one of two *disjuncts* expressed (the exclusive interpretation). The other asserts the truth of at least one the *disjuncts*, and allows for both *disjuncts* to be true (the inclusive interpretation).

V. Learning Goals: Following this lesson, the students should be able to:

1.) identify the form of the disjunctive proposition.
2.) construct disjunctive propositions out of other propositions.
3.) state the conditions for the truth of the disjunctive proposition.

Answer Key:

Exercise 1

Disjunction: "*p* or *q*"

Exercise 2

Answers will vary widely, however the compound propositions must follow the basic form guidelines for the kind of proposition discussed above: "*p* or *q*". Explanations should follow the guidelines about what it means for a disjunctive proposition to use an exclusive "or" versus and inclusive "or."

Exercise 3

1.

p	***q***	**"*p* or* *q*"**
True	True	False
False	True	True
True	False	True
False	False	False

2.

p	***q***	**"*p* or *q*"**
True	True	True
False	True	True
True	False	True
False	False	False

3.

p	***q***	**"*p* and *q*"**
True	True	True
False	True	False

True	False	False
False	False	False

Exercise 4

1.) This is true under the inclusive disjunction, as both disjuncts are true.
2.) Both disjuncts are false, so the disjunction is false.
3.) This is true under the inclusive disjunction, as both disjuncts are true.
4.) Both disjuncts are false, so the disjunction is false.
5.) The first disjunct is true, though the second is false, so the disjunction is true.