Distributed Versus Not Distributed

I. Lesson

"And yet, if *both* subject and predicate are used in their fullest extension, the resulting proposition will be false. For, indeed, no affirmation at all could, in those circumstances, be true."

(Aristotle, *On Interpretation* 7, trans. Harold P. Cook, 1962)

1) Distribution

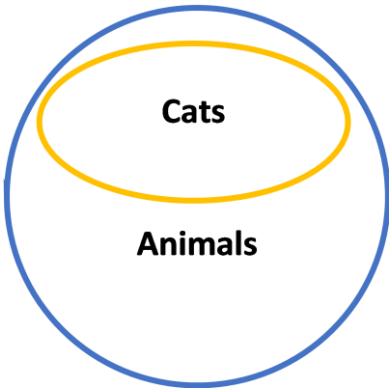
In a previous lesson, we discussed the helpful notion of *extension*. Recall that the extension of a term is just the totality of the members of the category the term refers to. For example, the extension of the term "tree" is the totality of all trees (since those are all the members of the category "tree"). Using this notion, we have the following two definitions, allowing us to classify terms in categorical propositions:

- **Distributed**: a term is *distributed* if we are using it to refer to its entire extension;
- **Not Distributed/Undistributed:** a term is *not distributed* or *undistributed* if we are using it to refer only to a part of its extension.

In the following, we will look at the distribution of terms – both subjects, and predicates – in each type of categorical sentence.

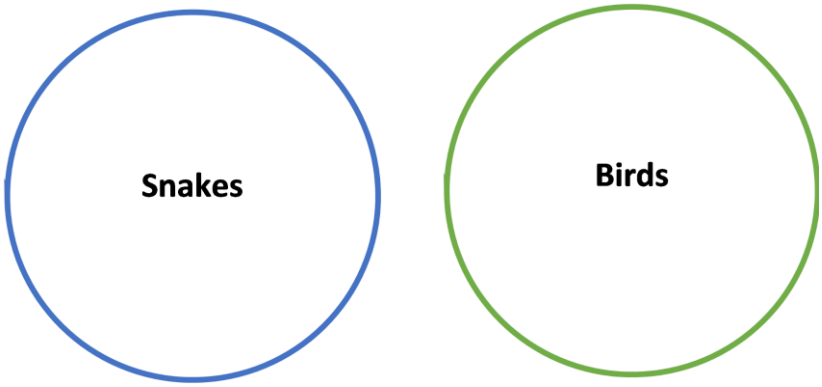
a) Universal Affirmative (UA)

Recall that Universal Affirmative (UA) propositions express that "All S is P." Let's look at our earlier example – "All cats are animals." In this case, we are talking about *all* cats; thus, in (UA) propositions, the subject term is always *distributed*; 'all' distributes over all the items of the subject term in (UA) propositions. The predicate, on the other hand, is not. When we express that all cats are animals, we are saying that every single cat qualifies as an animal and that *only some animals* qualify as cats. Through (UA) propositions, we are only referring to a *part* of the extension of the predicate.



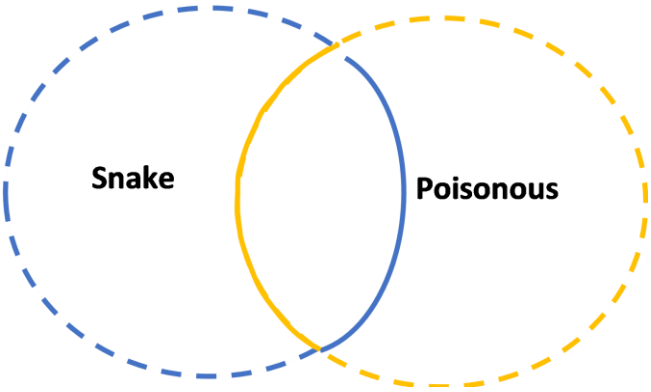
b) Universal Negative (UN)

Recall that Universal Negative (UN) propositions express that "No S is P." From our earlier lesson, consider the proposition "No snakes are birds." Again, we are referring to the entire extension of our subject. To see this more clearly, think of the proposition as saying that, of *all* the snakes, *none* of them qualify as a bird. Then, we observe that the subject term is again distributed; 'no' distributes over all the items of the subject term. In (UN) propositions, the predicate is also distributed. When we say of *all* the snakes that they are not birds, we also mean to say of *all* the birds that they are not snakes either.



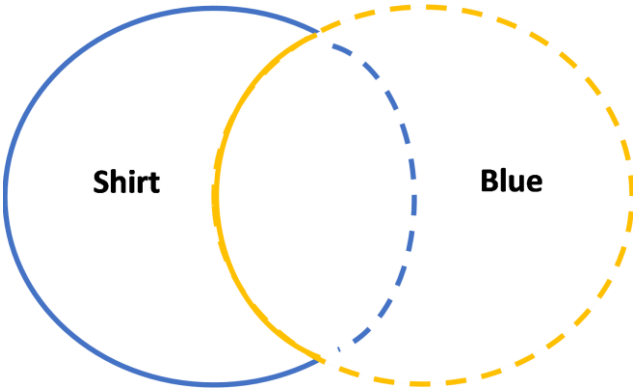
c) Particular Affirmative (PA)

Recall that Particular Affirmative (PA) propositions express that "Some S is P." One previous example was "Some snakes are poisonous." In this situation, neither the subject nor the predicate is distributed. In the case of the subject, this is fairly easy to see: we are using 'Some' to refer to the snakes, so we are only picking out a subset of all the snakes (quite simply, *some* of the snakes!). In the case of the predicate, we are also looking at a smaller subset of all the things that fall are 'poisonous', in particular those that also happen to be snakes. Clearly, not all poisonous things are snakes; only *some* of them are. Therefore, we are referring to *some* of the snakes and to *some* of the poisonous things, so both terms are not distributed.



d) Particular Negative (PN)

Recall that Particular Negative (PN) propositions express that "Some S is not P". In the previous lesson, we looked at "Some shirts are not blue" as an example. As with (PA) propositions, 'Some' ensures that we are only talking about a subset of all the shirts, so we know that the subject is not distributed. In the case of the predicate, we are taking the relevant subset of shirts and denying it *the entire category* of blue things. In other words, the shirts we are left with after subtracting the blue ones are shirts about which *no part* of the extension of 'blue' can be predicated. So the predicate is distributed.



[Activity] Draw a table with a partner representing the distribution of terms for each sentence type. As you do this, recall why each term is distributed or undistributed.

II. Examples

Distribution for (PA) and (PN) propositions can be quite confusing. Let's look at another kind of example for both in order to clear up any difficulties.

[Activity] Take an example (PA) proposition – "Some cats are albino." Say your friend says that because we know this, we know that "all cats have pale skin." But is your friend correctly inferring here? Discuss with a partner!

Let's take a different kind of example for (PN) propositions, using "Some snakes are not poisonous." Suppose we have 10 snakes and 10 poisonous things in our world. Of the 10 snakes, 7 are not poisonous, and 3 are poisonous. Then, we have 7 non-poisonous snakes, 3 poisonous snakes, and 7 non-snake poisonous things. When we say "Some snakes are not poisonous", we are taking the 7 snakes that are not poisonous and subtracting them from *the entire category* of poisonous things (snakes or otherwise). That is why the predicate is distributed, while the subject is not.

[Activity] How would you represent the above example visually? Think of using dots to represent the objects in the extensions of each term's category. Find a partner, share illustrations, and explain your choice of illustration.

[Activity] Can you find some other examples of your own? Asking yourself how different expressions we use commonly when we speak can be paraphrased as or correlated to logical forms can be a very helpful mechanism for a logician and a useful way to practice what you learn.

III. Exercises

For reference, consider the following table summarizing all our results regarding distribution for each sentence type:

	Subject Term	**Predicate Term**
Universal Affirmative (UA)	**Distributed**	**Not Distributed**
Universal Negative (UN)	**Distributed**	**Distributed**
Particular Affirmative (PA)	**Not Distributed**	**Not Distributed**
Particular Negative (PN)	**Not Distributed**	**Distributed**

Exercise 1

In the following propositions, identify the type of categorical proposition, the subject, and the predicate, and specify whether they are distributed or not distributed:

a) Some chairs are comfortable.

b) Not all numbers are odd.

c) Every cup of coffee is warm.

d) Of all the pastries, not a single one is disagreeable.

e) All logic problems are enjoyable.

f) Not all ideas are good.

Exercise 2

Draw Euler circles for the following statements:

a) Some glasses are colorful.

b) All chairs are comfortable.

c) Some trucks are not empty.

d) No pastry is disagreeable.

Exercise 3

Use the following 4 ordered ('subject', 'predicate') pairs to write (UA), (UN), (PA), and (PN) propositions, assigning exactly one pair to each type of proposition. Then, in your own words, explain the distribution of each term in each of the 4 propositions you have written.

a) (strawberry, red)

b) (houses, beautiful)

c) (fish, mammal)

d) (teaspoon, large)

Exercise 4

Look over the distribution for subjects and predicates in each type of proposition. What kind of subject is always distributed and what kind is always not distributed? What kind of predicate is always distributed and what kind is always not distributed?

Exercise 1

State what kind of proposition is described by the following:

a) A distributed subject and a distributed predicate.

b) A distributed subject and an undistributed predicate.

c) An undistributed subject and a distributed predicate.

d) An undistributed subject and an undistributed predicate.

IV. Conclusion

After having discussed each type of categorical proposition, we turned to look into one important feature of their terms, namely *distribution*, in order to better understand how each proposition relates the categories that the subject and the predicate refer to. When evaluating an argument using categorical propositions as its premises, as we will do in the future, distribution can be a powerful tool in helping us evaluate the truth of each premise. It simplifies propositional content to a series of inclusions and exclusions which is particularly easy to think about visually, too!

V. Lesson Goals

At the end of this lesson, students will be able to:

- define the notion of distribution with reference to the notion of intension;
- understand the distribution of terms in each of the 4 types of categorical propositions;
- justify the distribution of terms in each of the 4 types of categorical propositions;
- rephrase natural language variations on categorical propositions into their standard form in Aristotelian logic;
- represent and understand distribution through Euler circles;
- abstract the common features of subjects and predicates as they pertain to distribution through applying the notions of universal term, singular term, affirmative proposition, negative proposition.

Answer Key:

Exercise 1

a) (PA) proposition, (chairs, comfortable), both terms are not distributed;

b) (PN) proposition, (numbers, odd), subject is not distributed, predicate is distributed;

c) (UA) proposition, (cup of coffee, warm), subject is distributed, predicate is not distributed;

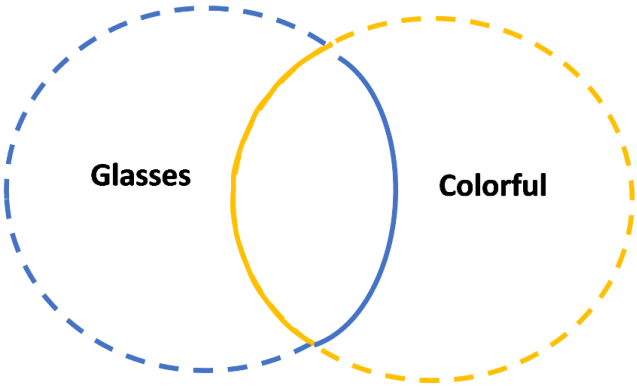
d) (UN) proposition, (pastries, disagreeable), both terms are distributed;

e) (UA) proposition, (logic problems, enjoyable), subject is distributed, predicate is not distributed;

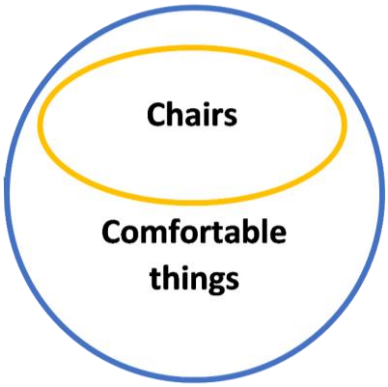
f) (PN) proposition, (ideas, good), subject is not distributed, predicate is distributed.

Exercise 2

(Circles do not need identical labels -- as long as it is clear what each category is)



a)



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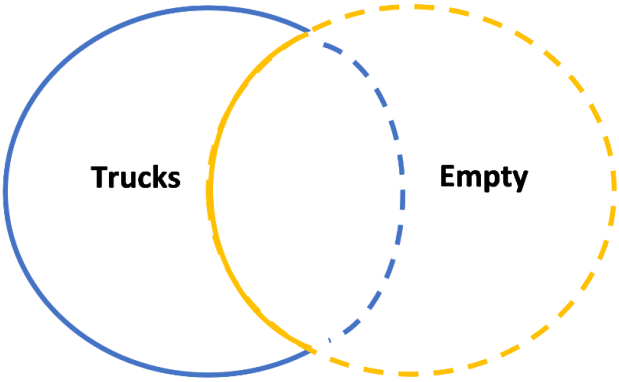

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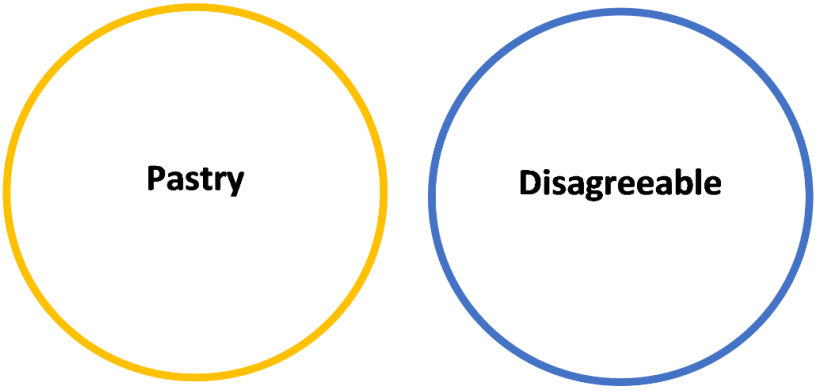

d)

Exercise 3

a) (as an example answer) "All strawberries are red" is an A sentence. Through 'all', we are referring to the entire extension of 'strawberry', so the subject is distributed. Since there are red things that are not strawberries, we are ranging only over a part of the extension of 'red', so the predicate is not distributed.

Exercise 4

Universal subjects are always distributed, while particular subjects are always not distributed. Predicates in negative propositions are always distributed, while predicates in affirmative propositions are always not distributed.

Exercise 5

a) (UN) proposition b) (UA) proposition c) (PN) proposition d) (PA) proposition