Compound Propositions: Conjunctive Propositions

I. Lesson

In many cases *categorical propositions*—for example, "All crocodiles are reptiles"—themselves may become elements of larger propositions. These larger propositions are known as *compound propositions*, and there are three types: *conjunctive* propositions, *disjunctive* propositions, and *hypothetical* propositions.

For the purposes of the next few lessons, we will use something known as *variable notation* to showcase the forms of these propositions. We briefly saw this when we represented the Universal Affirmative proposition, for instance, as "All *A*'s are *B*'s."

- **Variable Notation**: a short-hand notation in which we represent the instances of a proposition in an argument with variables (common examples of which are *p, q,* and *r*).

Just like math uses x or y to represent unknown quantities, logic sometimes uses variables to represent propositions. For example, we might represent all the instances of the proposition "Socrates was mortal" in an argument with the variable *p*, so that (1) we may more easily represent complex statements that involve that proposition and (2) we can more easily discern the formal qualities of that argument. Let's also represent "All men are mortal" as *q*, and "Socrates was a man" as *r*. Thus, instead of "If all men are mortal and Socrates was a man, then Socrates was mortal," we could represent this statement more succinctly as "If *q* and *r*, then *p*." In the following lessons, we will see why this kind of notation is useful.

Within this lesson, we'll deal with the first of the three kinds of compound propositions: *conjunctive* propositions.

- **Conjunctive Propositions**: a proposition consisting of two (or more) categorical propositions, linked together by "and." Sticking to the case of two categorical propositions, the logical form of such a proposition, in variable notation, is "*p* and *q*."

Conjunctive propositions are formed through a process known as conjunction, in which two propositions are linked together by "and." The conjunction of the two propositions "*p*" and "*q*" will take the form "*p* and *q*" in variable notation. Take the two propositions "A frog is an amphibian" and "A crocodile is a reptile." The conjunction of these two propositions —the conjunctive compound proposition — would be "A frog is an amphibian and a crocodile is a reptile."

Truth Conditions for Conjunctive Propositions: In determining the truth of a compound proposition, we have to consider the truth of each proposition in the compound. We call these propositions linked by "and" the *conjuncts* of a conjunctive proposition. For instance, if we have the conjunctive proposition, "Tables are pieces of furniture and all dogs are friendly," the two *conjuncts* would be (i) "Tables are pieces of furniture" and (ii) "all dogs are friendly." For the conjunctive proposition "and," the entire proposition is true if and only if both of the propositions within it are true. In other words, "*p* and *q*" is true if and only if *p* is true and *q* is true. For example, the proposition "a frog is an amphibian and a crocodile is a reptile" is true because "a frog is an amphibian" is true *and* "a crocodile is a reptile" is true. If either of the two *conjuncts* turns out false, the conjunction as a whole will be false. We can exactly how this works in the table below:

p	***q***	**"*p* and *q*"**
True	True	True
False	True	False
True	False	False
False	False	False

We can read this by saying (starting with the first row that assigns truth-values) "If *p* is true and *q* is true, then '*p* and *q*' is true." Secondly (now with the next row), "if *p* is false and *q* is true, then '*p* and *q*' is false." We can continue this for the fourth and fifth rows. As we see above, the only situation when the compound proposition "*p* and *q*" is true is when both *p* and *q* are independently true.

In our everyday usage of "and," we understand an "and"-sentence to be true only if both of its *conjuncts* are true. This holds for conjunctive propositions. If someone stated "1 + 1 = 3 and 3 + 3 = 6" or "1 + 1 = 2 and 2 +2 = 5," we would immediately say that person is wrong -- in the first utterance, because the first proposition was false; and in the second utterance, because the second proposition was false. But, if someone stated "1 + 1 = 2 and 2 + 2 = 4," we would call that statement correct, because *both* the propositions in the conjunctive statement are correct.

II. Examples

From our section above, let's look at the compound proposition: "A frog is an amphibian and a crocodile is a reptile", and try to piece together whether or not it is true.

This will be true, because both *conjuncts* themselves are true.

But suppose we switch the predicates and make new sentences with the same conjunctions, for example: "A frog is an amphibian and a crocodile is an amphibian" and "A frog is a reptile and a crocodile is an amphibian".

Both of these conjunctive propositions turn out to be false. The first is false, as its second conjunct is false. Despite the fact that the first of these two conjuncts is true, it must be remembered that the truth of a conjunctive proposition relies upon the truth of *both* of its conjuncts. The second is false too, as neither of its conjuncts are true.

III. Exercises

Exercise 1

Directions: Give the variable notation of the conjunctive proposition. Explain, in your own words, what variable notation is, and how it applies to the disjunctive proposition.

Exercise 2

Directions: Give three examples of true conjunctive propositions.

Ex. Pigs are animals and oaks are trees.

1.)
2.)
3.)

Exercise 3

Directions: fill out the following table describing whether the conjunctive complex proposition is

true. Try to do this from memory, not looking at tables from earlier in this lesson.

p	***q***	**"p and q"**
True	True	
False	True	
True	False	
False	False	

Exercise 4

Directions: Determine whether the following compound propositions are true or false, and justify your answer.

Ex. Birds have wings and pigs have wings.

Answer: False, as only the first of the two conjuncts is true, however the rules of conjunction state that only conjunctive propositions with two true conjuncts are true.

1.) Dogs are four-legged and dogs have four paws.
2.) The sky is blue and the sky is white.
3.) Balloons can float and chimpanzees are primates
4.) Trees are plants and trees are living.
5.) Aristotle was a logician and Plato was not a philosopher.

IV. Conclusion

Within Aristotelian logic, compound propositions are incredibly useful, as often we wish to assert more than one claim in the same proposition. With respect to conjunctions, Aristotelian logic is entirely in tune with our everyday intuitions: a conjunction is only true if both of its conjuncts are true. In this lesson, we discussed the conjunctive proposition, which claims to assert the truth of two propositions expressed.

V. Learning Goals: Following this lesson, the students should be able to:

1.) identify the form of the conjunctive proposition.
2.) construct conjunctive propositions out of other propositions.
3.) state the conditions for the truth of the conjunctive proposition.

Answer Key:

Exercise 1

Conjunction: "*p* and *q*"

Exercise 2

Answers will vary widely, however the compound propositions must follow the basic form guidelines for the kind of proposition discussed above: "*p* and *q*".

Exercise 3

p	***q***	**"*p* and *q*"**
True	True	True
False	True	False
True	False	False
False	False	False

Exercise 4

1.) This is true as both conjuncts are true.
2.) Generally, this would be considered false, as only the first of the two propositions is true, however if you interpret the clouds to be a part of the sky, then both propositions are true, so the whole conjunction is true.
3.) Both sides of the conjunction are true, so the proposition as a whole is true.
4.) Both sides of the conjunction are true, so the proposition as a whole is true.
5.) The first proposition is true, though the second is false, so the conjunction is false.