Extension and Intension

I. Lesson

A helpful method of understanding *categorematic* terms is the use of the concepts of **extension** and **intension**. To review once again, a *categorematic* term is a term that refers to a *category*. A *category* is just a collection of particular things that share some relevant common property or properties. This common property is referred to by the categorematic term itself. For instance, the *term* "chairs" picks out the *category* "chairs", which is just a collection of all of the things in existence that have the *property* of being a chair. All of those chairs are *members* of the category "chairs". It is worth noting that we can label a category in the singular or the plural. To be specific, "chair" and "chairs" are the same category: they have all things in reality that are chairs as members. Now we can understand the following definitions:

- **Extension of a categorematic term:** The totality of all the particular members of the category that the term refers to.
- **Intension of a categorematic term:** The meaning/definition of the term; The description of the common property/properties that all members of the extension of the term share.

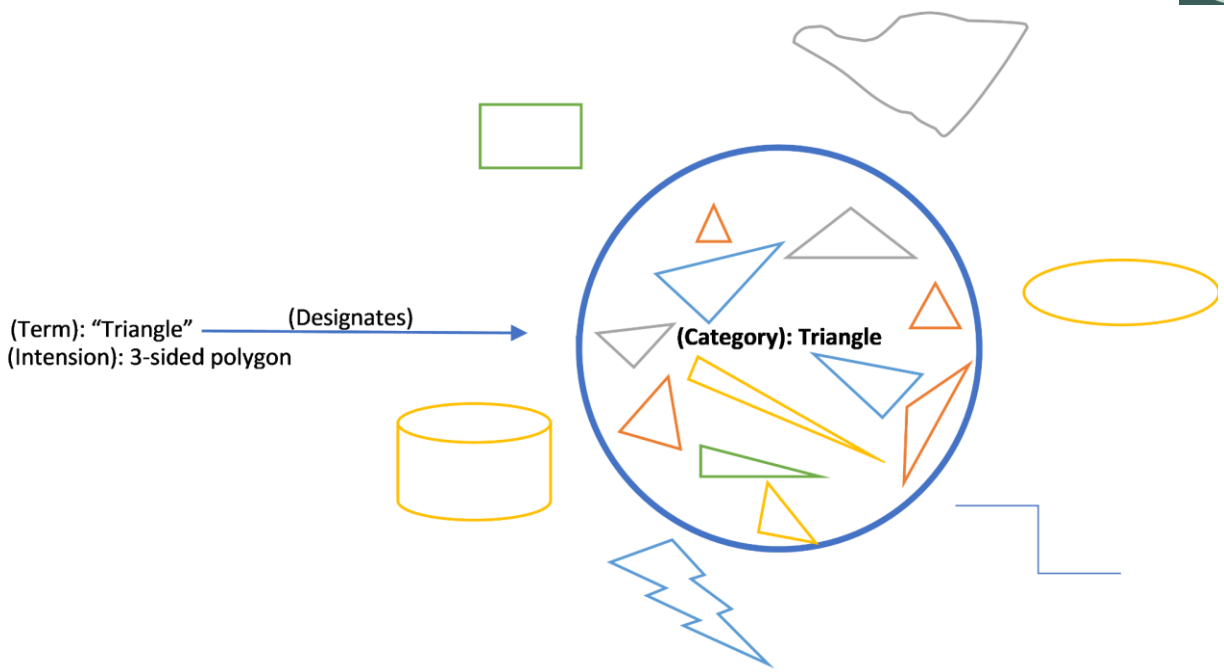
It's worth noting that the *intension* of a term can also be thought of as its *definition*. We can use the two terms interchangeably. Let's use an example to more easily understand these ideas. Let's say we have the term "triangle." The *intension*, or definition, of the term "triangle" is "a three-sided polygon". The *extension* of the term "triangle", then, is the totality of the things in reality that have the common properties of being both:

1. three-sided

   and

2. a polygon (straight sides, closed, and planar)

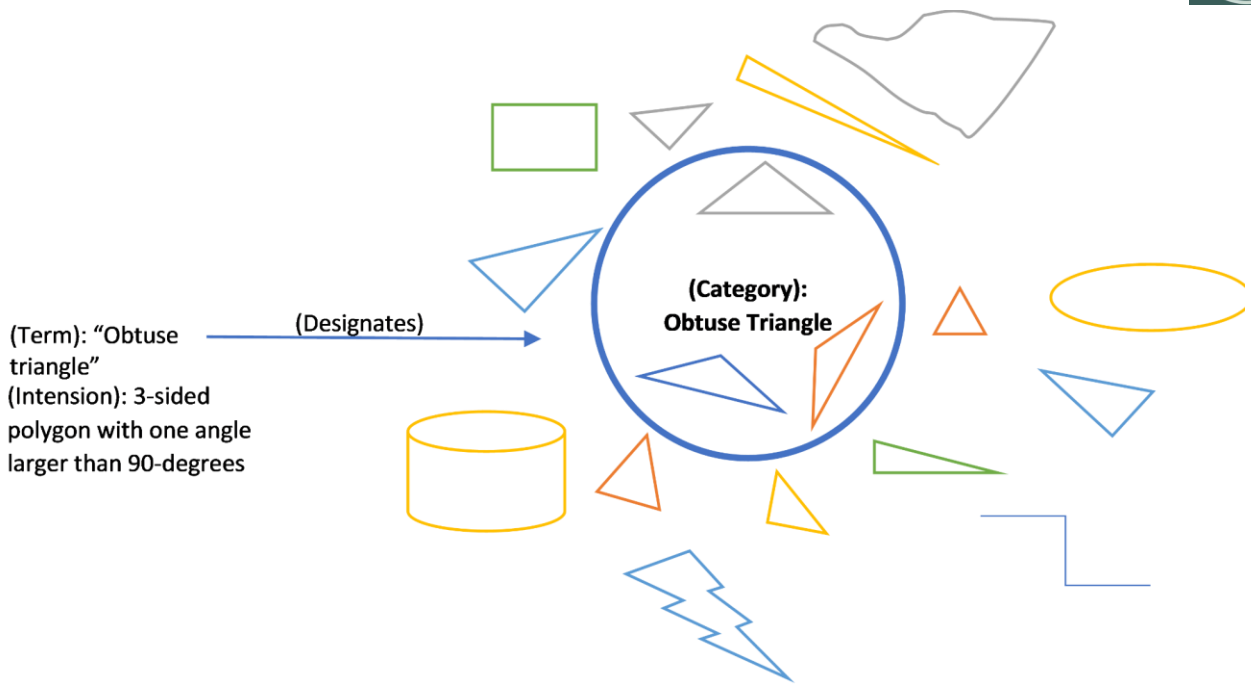
If there is something that is both (1) three-sided and (2) a polygon, then it is in the extension of "triangle." In other words, such a thing is a *member* of the category "triangle", since it has both of the common properties that *define* such a category. It is important to note that "define" comes from the latin "de" + "finire," or "to bound/limit completely". So, what *defines* the limit of a category? The definition/intension of the term that refers to it. And what things are in such a category? Only those things that have the common property/properties that a term's intension *defines* of them. Here's a picture to help visualize this:



As we can see, when we use the term "triangle", we refer to a category that contains the totality of things we call "triangle" (this totality is the extension of the term "triangle"). What objects are in this extension is *defined* by the intension of "triangle" given. These are the common properties held by all things *inside* the category but are not held by anything *outside* the category. In other words, everything inside the circle, or everything in the *extension* of the term, (1) has three sides and (2) is a polygon. Everything outside of the circle either does not have three sides, is not a polygon, or is neither a polygon nor three-sided.

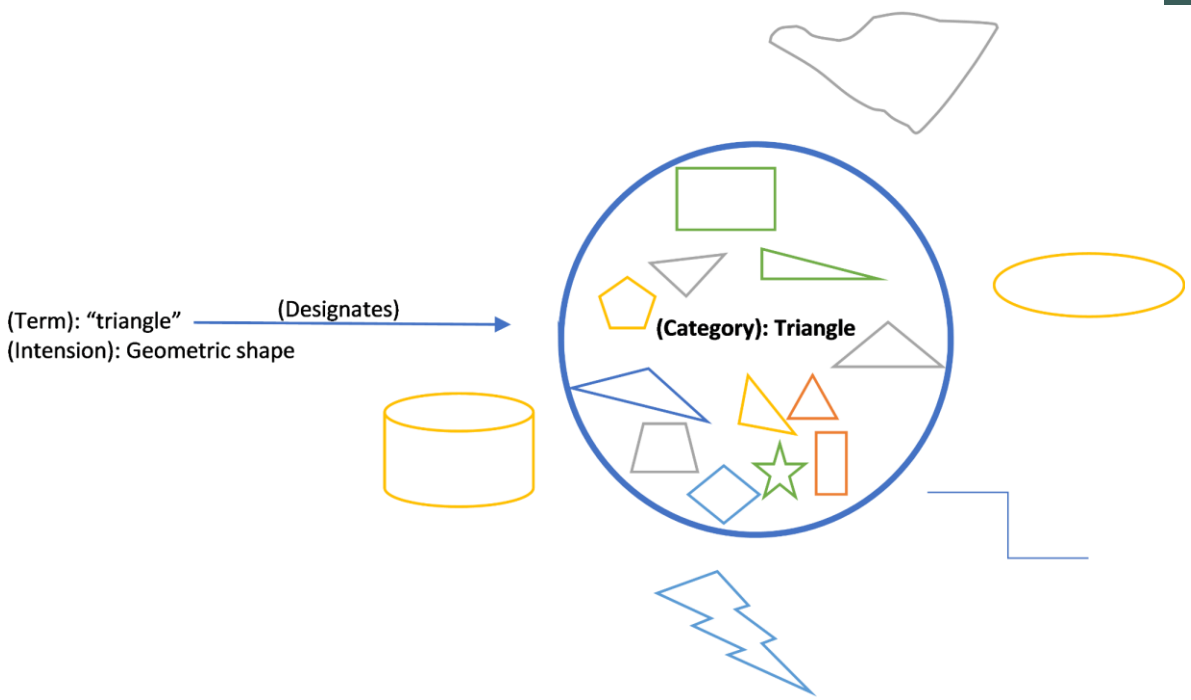
It is important to note that these two concepts are, in a way, *inversely related*. As the extension of a term *increases*, the term's definition must cover a wider range of things: its intension must become *less* specific. Similarly, as the extension of a term decreases, the term's definition covers a narrower range of things: its intension becomes *more* specific. As a general rule, the greater number of things to which a term refers, the less specific in meaning it must be.

For instance, consider an obtuse triangle. "Obtuse triangle" is a more specific term than "triangle" (it not only is a three-sided polygon, but it also has an angle larger than 90°). Thus, the intension has become more specific than that of a simple triangle. In other words, things must have *more properties* to be in the category "obtuse triangle". However, because the meaning of the term has become more specific, the collection of things that the term refers to in the world decreases. This general relation holds for almost all terms. We now have:



Notice that the category has become much smaller, because that which is limiting it (the intension) is *more specific*. There are now triangles outside of the category, because they are not *obtuse* triangles.

Intension is very important because it plays a crucial role in our definitions. When we define something, we want to choose a definition that is as specific as possible to fit the intuitive extension we associate with a term. Why is this the case? Take the triangle example again. If we were to define "triangle" with a definition that is broader than the generally-accepted intension of triangle, then we would have non-triangles in the category of "triangle." For instance, if we defined "triangle" as "a geometric shape," then squares would be members of the category "triangle," because squares have the property of being geometric shapes! Our bad intension would yield something like the diagram below:



This diagram shows that our definition/intension is *way* too broad, because now squares, stars, and trapezoids are in the category "triangle". Therefore, we know that we need to make our definition/intension more specific. Since our goal in argumentation is to avoid as much vagueness and ambiguity as possible, we want to make sure that our definitions include only the objects we are talking about. Of course, we also do not want our definition to be too specific either. We would like *all* and *only* triangles inside the category "triangle".

[Activity 1] With a partner, play a game that demonstrates the ideas of extension and intension. Have your partner define an intension. Now you must point to the extension of the intension defined. For instance, your partner might define the intension, "things which have four legs." Now, you might point to all of the chairs and desks (if they all have four legs). Once you do this with your partner a few times, switch roles: you define an intension and your partner will point to all the things that fall within the relevant extension.

[Activity 2] Now have your partner define an extension by pointing to certain things in the room. Based on what your partner chooses, come up with an intension to describe what was chosen. For instance, if your partner points only to chairs, then your job is easy! You would simply label the intension as "chair." Now, if your partner points to pencils, paper, rulers, etc., then you might define the intension as "school supplies." Try to define an intension that is as specific as possible. For instance, in the previous example, say something like "school supplies" rather than "things." Once you do this with your partner a few times, switch roles: you define an extension and your partner will define an intension that fits that extension.

I. Examples

(1) First, take the term "human." There are many ways to define what it means to be human, but perhaps you agree with Aristotle and define a human as "a rational animal." This is the intension of the term. The extension of the term is the totality of the rational animals (humans) which exist in reality. We see that we may have made our definition too broad, and that there exist other rational animals besides humans. Let's, however, accept the idea that "a rational animal" is a good definition for "human." Let's make the term more specific and see what happens to the extension and intension of the term. Perhaps we choose to specify by using the term "female human." Now the intension of the term increases in terms of specificity, since the new meaning of the term is "a female rational animal." Conversely, the extension decreases, since the number of female humans in reality is fewer than the number of total humans.

(2) Next, let's choose a more abstract term. Let's say we have the term "justice." Although justice is notoriously difficult to define, let's say that we accept the definition, : "justice is the interest of the stronger" (this is the definition was put forth by Thrasymachus, an interlocutor in Plato's foundational work of political philosophy, *Republic*). Even though this is an abstract term, we can see that the same inverse principle between extension and intension applies. For instance, we can modify Thrasymachus's definition and say that "justice" is "the interest of both the stronger and the weaker at the same time." We can intuitively see that this new definition would occur fewer times in the world (a smaller extension), because there are few instances in which both the stronger and the weaker benefit.

II. Exercises

1. For each of the following collections of things, stipulate a term with an intension such that the collection becomes the extension of the intension defined. Choose as specific an intension as possible.

Ex. Oak, Pine, Maple, Cedar

Best Answer: "Tree" (all of the things above are part of the extension of the standard intension of "tree.")

Worse Answer: "Plant" (while it is true that all of those things are plants, this is not the most specific intension).

a.) California, Michigan, Kentucky, Pennsylvania

b.) Monday, Tuesday, Saturday
c.) Apples, bananas, mangos, oranges
d.) California, Paris, Yellowstone National Park, Russia
e.) Chairs, tables, cabinets, dressers
f.) Mars, the Sun, Neptune, Earth
g.) Rutgers University, New York University, Columbia University
h.) Rutgers University, Frederick Douglass Elementary School, Allentown High School

2. For each of the intensions given below, list 5 things that would fit the relevant extension.

Ex. Companies

Answer (could vary): Nike, McDonald's, Target, Hackett Publishing Company, Facebook

a.) People in your classroom
b.) Holidays which take place between the beginning of January and the end of July
c.) Presidents of the United States
d.) Polygons which have more than 3 sides
e.) Polygons which have fewer than eight sides
f.) Subjects taught in your school
g.) Places
h.) Places in the United States
i.) Places in your city

Notice: if you answered questions (a)-(i) correctly, you will see that the extensions are decreasing as the intension becomes more specific.

3. Make a case for why the following definitions (intensions) are too broad or too narrow.

Ex. Lions – large cats

Answer: This definition is too broad. There are many large cats that we would not consider lions (tigers or cougars for instance). Thus, there are members of the extension of "large cats" that are not lions – the definition is too broad.

a.) Humans – things that at a given moment are featherless bipeds (2 legged animals that have no feathers)
b.) Humans – rational featherless bipeds that weigh more than 50 pounds
c.) Squares – four-sided polygons

d.) Squares – four-sided polygons with equal side lengths and four right angles, where each side is longer than 10 inches
e.) Mathematics – a boring thing to study
f.) Sandwich – a type of food that has ham, cheese, and other things between two pieces of bread
g.) School – a place where someone eats lunch, takes some classes, learns from teachers, and brings a backpack
h.) Aristotle – A philosopher from Ancient Greece

4. Evaluate each of the following arguments, and write whether or not the terms being used have been defined correctly, or if their definitions are too broad or too narrow.

a.) Humans can be defined as "animals."
Some animals have four legs.
Therefore, some humans have four legs.
b.) Triangles can be defined as "three-sided polygons."
Each side face of the pyramids was roughly a three-sided polygon.
Therefore, each side face of the pyramids was roughly a triangle.
c.) Computers can be defined as "electronic devices."
Power tools are electronic devices .
Therefore, power tools are computers.
d.) Countries can be defined as "areas of land greater than 50,000 square miles with a legitimate government."
Austria has a land area less than 50,000 square miles.
Therefore, Austria is not a country.

5. Distinguish these four terms:
a.) Categorematic Term
b.) Category
c.) Intension
d.) Extension

III. Conclusion

The notions of extension and intension are incredibly important because they help us make sure that we define (categorematic) terms correctly in any given circumstance. In an argument, it is crucial that we understand intension and extension so that our terms are clear and unambiguous. When we evaluate arguments, it's important to examine all the terms that are being used, and make sure that such terms are defined correctly. If terms are given a definition that is too broad or too narrow, it is

easy to see that an absurd conclusion could follow. After all, since syllogisms are composed of propositions, and propositions are composed of terms, if the terms are faulty, then the entire argument could easily go wrong. Now that we know how terms relate to categories, we can begin relating categories themselves in *propositions*.

IV. Lesson Goals

At the end of this lesson, students will be able to:

- Understand the difference between extension and intension
- Understand how extension and intension are inversely related
- Understand how notions of extension and intension inform our definitions of things
- Understand why a definition is too broad or too narrow
- Understand why some arguments fail on the basis of having ill-defined terms
- Understand the difference between extension and category

Answer Key:

Exercise 1

a.) "States" (with the term's standard intension)
b.) "Days of the week" (with the term's standard intension)
c.) "Fruits" (with the term's standard intension)
d.) "Places" (with the term's standard intension)
e.) "Pieces of furniture" (with the term's standard intension)
f.) "Solar Bodies" (with the term's standard intension)
g.) "Universities" (with the term's standard intension)
h.) "Schools" (with the term's standard intension)

Exercise 2

a.) (Answers may vary.) Correct answers are names of people in the classroom at the moment
b.) (Answers may vary.) New Year's Day, MLK Day, Groundhog Day, Valentines Day, Easter Independence Day, etc.
c.) (Answers may vary.) George Washington, Abraham Lincoln, Theodore Roosevelt, Ronald Reagan, Barack Obama, etc.
d.) (Answers may vary.) Quadrilateral, Square, Rectangle, Pentagon, Hexagon, Heptagon, Octagon, etc.
e.) (Answers may vary.) Heptagon, Hexagon, Pentagon, Rectangle, Square, Trapezoid, Triangle, etc.
f.) (Answers may vary.) Math, English, Social Studies, Chemistry, Physics, etc.
g.) (Answers may vary.) My front yard, London, California, Mars, down the hall, etc.
h.) (Answers may vary.) New York, Denver, Appalachia, down the hall, Yellowstone National Park, etc.
i.) (Answers may vary depending on the city.)

Exercise 3

a.) Definition is too broad. If anything that was a featherless biped at a certain time was a human, then a plucked chicken would be a human (Diogenes' counterexample to Plato's definition of "man").
b.) Definition is too narrow. Small children are under 50 pounds, and they are still human.
c.) Definition is too broad. There are many four-sided polygons that are not squares – many instances of rectangles, trapezoids, parallelograms, etc.

d.) Definition is too narrow. There certainly are squares with sides shorter than 10 inches.
e.) Definition is too broad. Ostensibly, there are boring things to study that aren't math. For instance, most people would find an in-depth study of the back of one's hand boring.
f.) Definition is too narrow. There are definitely types of food that have just cheese (no ham) between two pieces of bread that would be considered sandwiches. Under this definition, grilled cheese sandwiches would not be sandwiches.
g.) Definition is too narrow. If you don't bring your backpack to school, it's still a school. Likewise, schools don't have to offer a place or time to eat lunch to be considered a school.
h.) Definition is too broad. There were many philosophers from Ancient Greece besides Aristotle.

Exercise 4

a.) The definition of "humans" given here is far too broad. There are many animals that are not humans.
b.) The terms are defined correctly here. As such, this is a sound argument.
c.) The definition of "computer" here is too broad. There are many types of electronic devices that are not computers.
d.) The definition of "country" here is too narrow. There are many countries which have a square area of less than 50,000 square miles.

Exercise 5

A categorematic term is simply a linguistic expression that can be neither true nor false and refers to a category. A category is a collection of particular things that share a common relevant property which is exclusive to those members. The intension of the term is a description of this common property that all members of the category share. The extension of the term is the totality of all of the members of the category in question.