Modus Ponens and Modus Tollens

I. Lesson

1. Key Definitions (Review):

- **Hypothetical Proposition:** a type of compound proposition in which two propositions are linked through the assertion that the truth of the *antecedent* implies the truth of the *consequent*.
  - The form of this proposition is "If *p*, then *q*".
  - Ex: "If it snows in New York, then the temperature in New York is less than 32 degrees Celsius".

- **Antecedent:** the initial proposition of the pair which appears in the hypothetical.
  - In the above example, the antecedent is "it snows in New York".

- **Consequent:** the final proposition of the pair which appear in the hypothetical
  - In the above example, the antecedent is "the temperature in New York is less than 32 degrees Fahrenheit".

2. Modus Ponens and Modus Tollens for Hypothetical Propositions:

(i) Modus Ponens: A rule of inference which states that if a hypothetical, "If *p*, then *q*", is true, and the antecedent "*p*" is true, then the consequent, "*q*" is true. In variable notation, the rule appears in the form:

(1) If *p*, then *q*
(2) *p*
(3) Therefore, *q*

(ii) Modus Tollens: A rule of inference which states that if a hypothetical, "If *p*, then *q*", is true, and the consequent "*q*" is false, then the antecedent "*p*" is false. In variable notation, the rule appears in the form:

(1) If *p*, then *q*
(2) not-*q*
(3) Therefore, not *p*

Explanatory Note: The hypothetical informs us that the truth of *p* implies the truth of *q*. This being the case, if *p* isn't true, then *q* can't be true either, due to the fact that the hypothetical links the truth

of *p* to that of *q* (Modus Ponens). On the other hand, if *q* isn't true, since the truth of *p* ensures the truth of *q*, then *p* can't be true (Modus Tollens).

3. Modus Ponens and Modus Tollens again, but with <u>Categorical Propositions</u>

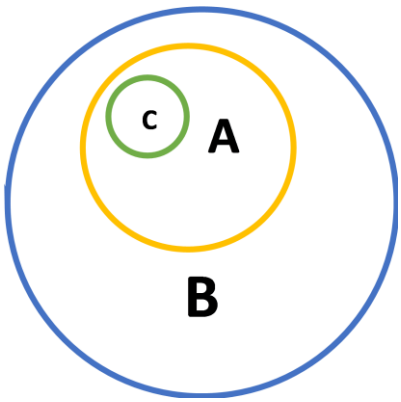
(i) Modus Ponens: This rule occurs when (a) every member of a certain category *A* is a member of another category *B*, and (b) that some thing(s) *C* is/are member(s) of category *A*. It follows that *C* is/are member(s) of category *B*.

(1) All *A*'s are *B*'s
(2) *C* is an *A* / All *C*'s are *A*'s
(3) Therefore, *C* is a *B* / All *C*'s are *B*'s

(ii) Modus Tollens: This rule occurs when (a) every member of a certain category A is a member of another category B, and (b) that some thing(s) C is/are not members of category B. It follows that C is/are *not* member(s) of category A.

(1) All *A*'s are *B*'s
(2) *C* is not a *B* / No *C*'s are *B*'s
(3) Therefore, *C* is not an *A* / No *C*'s are *A*'s.

It becomes clear that these versions of Modus Ponens and Modus Tollens are quite similar to the ones introduced in section **2.** "All *A*'s are *B*'s," means "If something is an *A*, then it is a *B*." For Modus Ponens, if something specific, *C*, is an *A*, it follows that *C* is a *B*. We can also visualize these with Euler circles. Here is Modus Ponens:



As one can see, all *A*'s are *B*'s here, and all *C*'s are *A*'s. It follows that all *C*'s are *B*'s! Now, here is Modus Tollens:

Here, we see that all *A*'s are *B*'s, and that no *C*'s are *B*'s. It follows that no *C*'s are *A*'s!

II. Examples

Within the following section, we will be looking through a couple of examples depicting uses of Modus Ponens and Modus Tollens.

Activity: Take the two short paragraphs, which each represent an example of the law of Modus Ponens. In each case, identify the hypothetical compound proposition, and determine which portion is the antecedent, and which is the consequent.

i. Modus Ponens in the real world

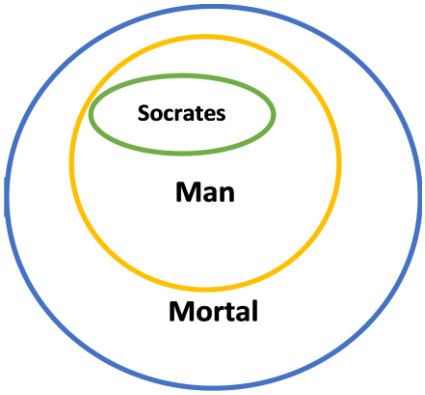
1. Suppose I know that if my dog sees a squirrel, then it will chase that squirrel. Now, suppose that I can see that my dog is looking at a squirrel. I will infer from this that my dog will chase the squirrel. This inference is an example of Modus Ponens.

2. Suppose that the book, *Crime and Punishment* by Fyodor Dostoevsky is on your favorite bookshelf in your room. Your brother walks into your room and says that all of the books on your favorite bookshelf are some of his favorite books. You infer (by Modus Ponens) that *Crime and Punishment* is one of your brother's favorite books.

ii. Modus Ponens in the form of a syllogism

1. (1) If Johnny is a lawyer, then Johnny studies the law *(if p, then q)*
   (2) Johnny is a lawyer *(p)*
   (3) Therefore, Johnny studies the law *(therefore, q)*

2. (1) All men are mortal *(All A's are B's)*
   (2) Socrates is a man *(C is an A)*
   (3) Therefore, Socrates is mortal *(C is a B)*

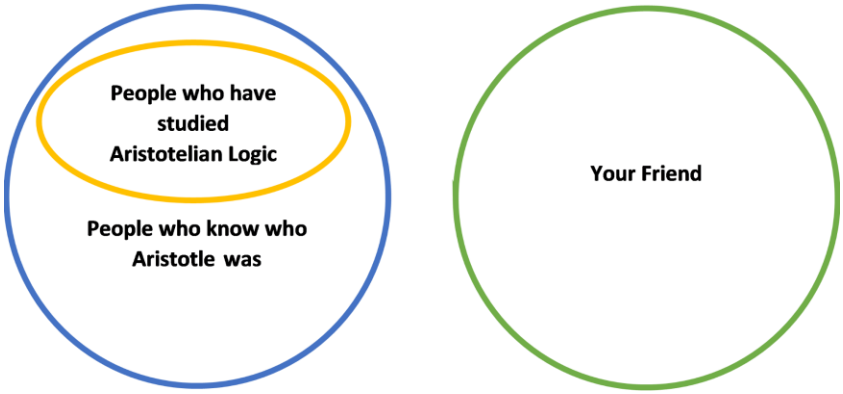


Activity: Come up with 2-3 real world examples of reasoning using Modus Ponens; discuss these with a classmate to make sure they are actually representative. Try to put these examples into the form of a syllogism.

iii. Modus Tollens in the real world.

1. As from before, suppose that I know that if my dog sees a squirrel, then it will chase that squirrel. Now, suppose that I know that my dog has not chased any squirrels today. I will infer from this that my dog hasn't seen any squirrels today. This inference is an example of Modus Tollens.

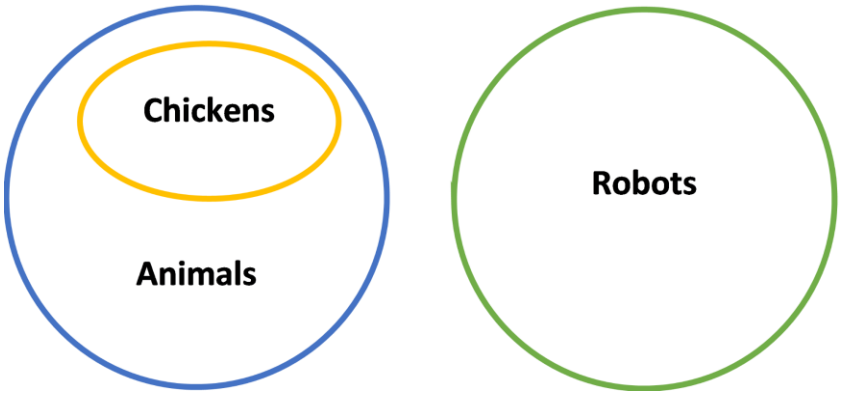
2. Suppose you know that any person who has studied Aristotelian Logic would know who Aristotle was. You ask your friend if they know about Aristotle, and she says that she does not. You infer that she has not studied Aristotelian Logic.



iv. Modus Ponens in the form of a syllogism

1. (1) If Newton was right about everything, then he would have no false theories *(if p, then q)*
   (2) He has false theories *(not-q)*
   (3) Therefore, Newton was not right about everything *(therefore, not-p)*

2. (1) All chickens are animals *(All A's are B's)*
   (2) No robots are animals *(C is not a B)*
   (3) Therefore, robots are not chickens *(therefore, C is not an A)*



Activity: As before, come up with 2-3 real world examples of reasoning using Modus Tollens; discuss these with a classmate to make sure they are actually representative. Try to put these examples into the form of a syllogism.

III. Exercises

Exercise 1

Directions: Answer the following questions

1.) What is the structure of arguments using the rule of Modus Ponens? (Write this structure using variable notation)
2.) What is the structure of arguments using the rule of Modus Tollens? (Write this structure using variable notation)
3.) What are the two parts of the hypothetical? (Differentiate between these two)
4.) What does it mean to say that Modus Ponens and Modus Tollens are *valid* rules of inference?

Exercise 2

Directions: Determine whether the following arguments are examples of Modus Ponens, Modus Tollens, or neither.

Ex. I know that if birds can fly, then they must have wings. I know that penguins don't have wings, therefore I know that they can't fly.

Answer: Modus Tollens

1.) I know that if a tree is sturdy, it has very deep roots. I know the tree in my front yard is sturdy, so I know it must have deep roots.
2.) I know that if my friend gets a pet, I will be able to play with it. I recently have not been able to play with any animals, so I know my friend must have lost their pet.
3.) I know that if someone stares at their computer too long their eyes will hurt. I know that my eyes feel okay, so I know that I must have not been staring at my computer too long.

Exercise 3

Directions: Determine whether the following arguments are Modus Ponens, Modus Tollens, or neither. Secondly, if they are an example of Modus Ponens or Modus Tollens, determine whether they deal with hypothetical propositions or only categorical propositions.

Ex. (1) Aristotle is a philosopher.
(2) All philosophers are lovers of wisdom.

(3) Therefore, Aristotle is a lover of wisdom.

Answer: Modus Ponens, only categorical propositions (*remember:* the order of the premises *does not matter*!)

1.) (1) If you are going to the store, I will not be able to go to my friend's house.
(2) I will be able to go to my friend's house.
(3) Therefore, you are not going to the store.

2.) (1) All soccer games are fun to watch.
(2) What is on the television right now is not fun to watch.
(3) Therefore, what is on the television right now is not a soccer game.

3.) (1) Aristotle is the greatest philosopher to ever live.
(2) Plato is the greatest philosopher to ever live.
(3) Therefore, Aristotle is Plato.

4.) (1) Aristotle was not appointed by Plato as Plato's heir.
(2) If Aristotle was Plato's greatest student, then Plato would have appointed Aristotle as his heir.
(3) Therefore, Aristotle was not Plato's greatest student.

5.) (1) The speed of light is constant.
(2) If the speed of light is constant, then Einstein was right about something.
(3) Therefore, Einstein was right about something.

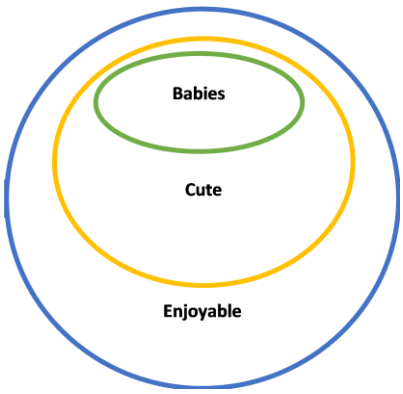
6.) (1) Either the speed of light is constant, or the speed of light can vary.
(2) The speed of light cannot vary.
(3) Therefore, the speed of light is constant

7.) (1) If logic is not helpful, then you should not learn logic.
(2) You should learn logic!
(3) Therefore, logic is helpful.

Exercise 4

Directions: Look at the following Euler circles and write out the argument in syllogistic form.

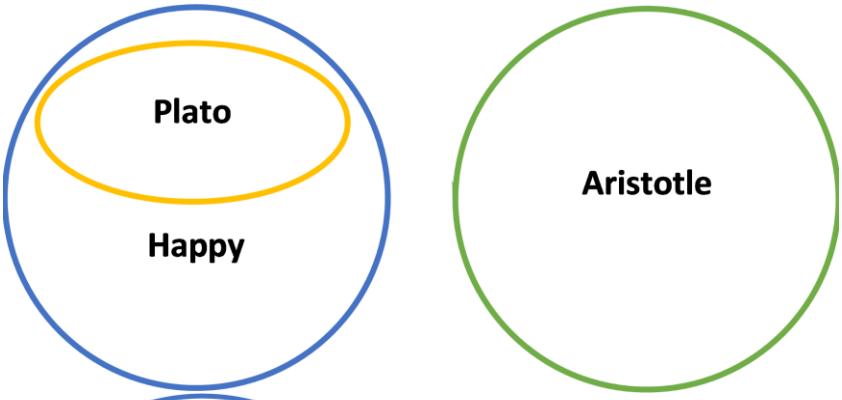
Ex.



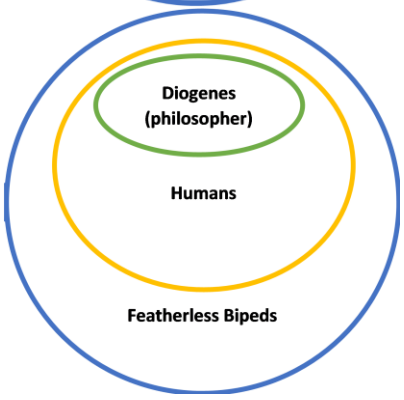
Answer: (1) All babies are cute
(2) All cute things are enjoyable
(3) Therefore, all babies are enjoyable

Remember, the order of the premises does not matter, and the premises *do not need to be true*.

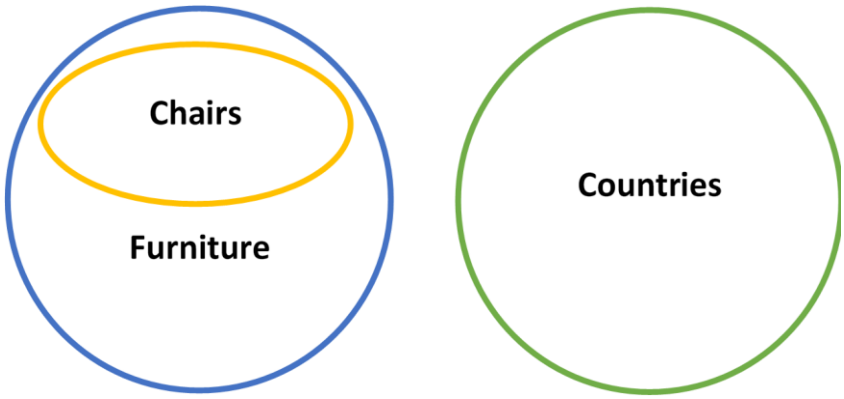
1.)



2.)



3.)



Exercise 5 (Challenge Question)

Directions: Come up with your own arguments utilizing the rules of Modus Ponens and Modus Tollens. Write down two arguments for each rule.

1.)

2.)

3.)

4.)

IV. Conclusion

Often in general debate, not just in logic, conditional statements are used. As the rules of Modus Ponens and Modus Tollens both can be used to provide valid arguments based on the supposition of a conditional, they are both incredibly useful within any form of argumentation or debate. In this lesson, we went over both of these rules in detail so that in the future, when conditional statements are used around you, you will have the tools to determine which conclusions can be drawn based on the statements. Furthermore, these rules of inference can be used with only categorical propositions. This allows for a concrete application of Universal Affirmative and Negative propositions from earlier lessons.

V. Learning Goals

Following this lesson, the students should be able to formulate and identify arguments following the rules of Modus Ponens and Modus Tollens. The student should also have a grasp of what a rule of inference is, as well as further understanding of the hypothetical compound proposition.

Answer Key:

Exercise 1

1. If *p*, then *q*; *p*; therefore *q*.
2. If *p* then *q*; not *q*; therefore not *p*.
3. The two parts of the hypothetical are the *antecedent* and the *consequent*. The antecedent entails the consequent. In other words, the antecedent appears first in the conditional (*p*), and the consequent appears second in the conditional (*q*), so that "if *p*, then *q*."
4. This is to say that if all of the premises are true, Modus Ponens and Modus Tollens guarantee the truth of the conclusion. In other words, they will always yield a true conclusion if the premises are true.

Exercise 2

1. Modus Ponens
2. Neither
3. Modus Tollens

Exercise 3

1. Modus Tollens, hypothetical proposition
2. Modus Tollens, only categorical propositions
3. Neither
4. Modus Tollens, hypothetical proposition
5. Modus Ponens, hypothetical proposition
6. Neither
7. Modus Tollens, hypothetical proposition

Exercise 4

(Premise order may vary)

1. (1) Plato is happy

   (2) Aristotle is not happy

   (3) Therefore, Aristotle is not Plato

2. (1) All humans are featherless bipeds.

   (2) Diogenes is a human.

   (3) Therefore, Diogenes is a featherless biped.

3. (1) All chairs are pieces of furniture.

   (2) No countries are pieces of furniture.

   (3) Therefore, no countries are chairs.

Exercise 5 (Challenge Question)

Answers will vary widely, however two of the arguments provided must follow the outline "If *p*, then *q*; *p*; therefore *q*", and the other two must follow the outline "If *p* then *q*; not *q*; therefore, not *p*".