The Square of Opposition

I. Lesson

We previously learned about the four different types of propositions. Now, we will get to observe some interesting logical relations that exist among them. The relations are as follows:

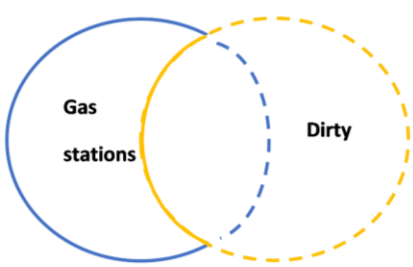
1. *Contradictory Relation:* when the truth of one proposition implies the falsity of the other, and vice versa, meaning that the two propositions can never both be true at the same time nor can both be false at the same time.
2. *Contrary Relation:* when the two propositions cannot both be true at the same time (although they can both be false).
3. *Subcontrary Relation:* when the two propositions cannot both be false at the same time (although they can both be true).
4. *Subalternation:* when the truth of the first proposition implies the truth of the second (but not the other way around).

II. Examples

The definitions above are great, but it's much easier to see what they really mean when we look at some examples. These examples also help us to see the different types of propositions that can be involved in each of the relations.

1. Contradictory Relation:

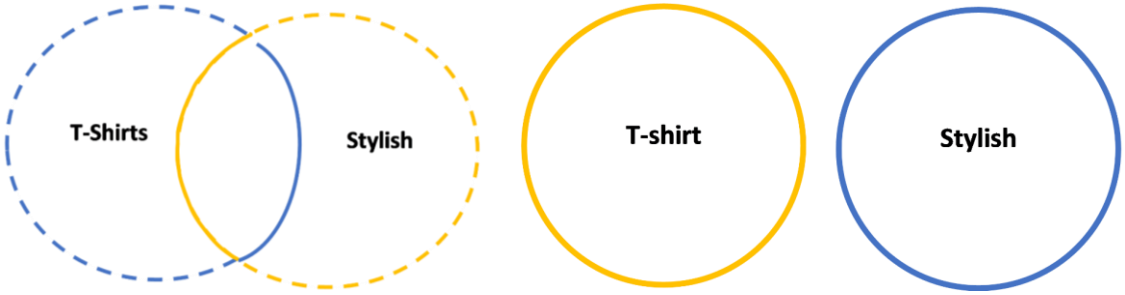
- This relation exists between Universal Affirmative and Particular Negative propositions. If "all gas stations are dirty" (UA) is true, then "some gas stations are not dirty" (PN) must be false. Conversely, if "some gas stations are not dirty" (PN) is true, then "all gas stations are dirty" (UA) must be false. Euler Circles will help us intuitively visualize this:



It is evident that one of these being true implies that the other is false. Suppose that what's represented by the right diagram (UA) is true. If (UA) is true, then *all members* of "gas stations" are also members of the category "dirty" (so no members of "gas stations" are not members of the category "dirty.") But then the left (PN) is explicitly false, since it claims there

are members of the category "Gas stations" which are 'outside' the category "dirty". Now, suppose that (UA) here is false. Then it is *not the case* that all members of "gas stations" are also members of the category "dirty." In other words, some members of "gas stations" must be '*outside*' of "dirty". This is explicitly what the left diagram states, so (PN) would be true.

The *contradictory relation* also holds between Universal Negative and Particular Affirmative propositions. If "no T-shirts are stylish" (UN) is true, then "some T-shirts are stylish" (PA) must be false. Conversely, if "some T-shirts are stylish"(PA) is true, then "no T-shirts are stylish" (UN) must be false.



Suppose the right diagram (UN) is true. Then there is no overlap between the categories "T-shirt" and "stylish." Then the left diagram (PA) is explicitly false, since it says there is such an overlap. Now, suppose that (UN) is false. Then the two categories are *not* completely separate (there are some objects that are members of *both* of them). But then (PA) is explicitly true.

2. *Contrary Relation:*

- This relation exists between Universal Affirmative and Universal Negative propositions. "All children are cute" (UA) and "no children are cute" (UN) cannot both be true. That said, both propositions can be false—it may be the case that some, but not all, children are cute!

Now it is clear that the left (UA) and the right (UN) cannot both be true at the same time. It can't both be the case that all of the category "children" is contained within "cute" *and* that the two categories are completely separate at the same time. However, both (UA) and (UN) can be

false at the same time. There could be overlap between the categories, which would make both (UA) and (UN) false.

3. *Subcontrary Relation:*

- This relation exists between Particular Affirmative and Particular Negative propositions. "Some candy bars are free" (PA) and "some candy bars are not free" (PN) cannot both be false. However, both propositions have the potential to be true (if we only lived in a more generous world than the one we actually live in!).

  We won't need diagrams for this one, since we can simply use the ones already given. Intuitively, both (PA) and (PN) can be true at the same time. This would just mean that the diagram would look different from either given in (2) above, even after all information is known. By what we learned in (1), if "some candy bars are free" (PA) is false, then "No candy bars are free" (UN) is true. Likewise, if "some candy bars are not free" (PN) is false, then "All candy bars are free" (UA) is true. But, by what we learned in (2), (UN) and (UA) cannot both be true! Therefore, (PA) and (PN) cannot both be false!

4. *Subalternation:*

- This relation exists between Universal Affirmative and Particular Affirmative propositions. "All chocolate desserts are delicious" (UA) being true implies that "some chocolate desserts are delicious"(PA) is also true. But be careful—if "some chocolate desserts are delicious" (PA) is true, it does not necessarily mean that "all chocolate desserts are delicious" (UA) is also true! This means that (PA) propositions are the subalterns of (UA) propositions, *but not vice versa*. In other words, (PA) propositions do not have subalterns.

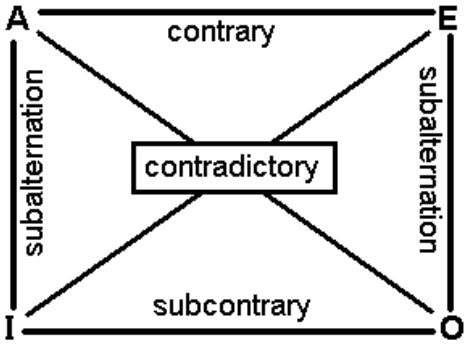
  This relation also exists between Universal Negative and Particular Negative propositions. "No teenagers are early risers" (UN) being true implies that "some teenagers are not early risers" (PN) is also true. However, "some teenagers are not early risers" (UN) being true does not imply that "no teenagers are early risers" (PN) is also true. This means that (PN) propositions are the subalterns of (UN) propositions, *but not vice versa*. In other words, (PN) propositions do not have subalterns. We therefore have the following:

  - (PN) is the subaltern of (UN)
  - (PA) is the subaltern of (UA)
  - Nothing is the subaltern of (PN)
  - Nothing is the subaltern of (PA)

III. A Very Useful Diagram

These relations can be pretty hard to keep track of. This is where the Square of Opposition, created in the Middle Ages but based on Aristotle's findings, comes in handy. This diagram allows us to take note of all the relations simultaneously.

- In the diagram, Universal Affirmative propositions are labeled with the letter "A." Universal Negative propositions are labeled with an "E." Particular Affirmative propositions are labeled with an "I." Particular Negative propositions are labeled with an "O." These are all standard conventions.
- The different lines connecting these letters indicate the four types of relations listed above. Remember, though, the subaltern relation is only *one way*.



IV. Exercises

Exercise 1

Directions: Consider the following propositions and answer the following questions: What type of proposition is this? What proposition would be its contrary? What proposition would be its subaltern? What proposition would be its contradiction? What proposition would be its subcontrary? (*hint*: some propositions, depending on the type, might not have subalterns or contraries or subcontraries. Use the square of opposition to help you.)

i. "All dogs are friendly."
ii. "Some houses are not expensive."
iii. "Some bumblebees are loud."
iv. "No philosophers are musicians"

Exercise 2

Directions: Describe in your own words why particular categorical propositions do not have subalterns.

Exercise 3 (Challenge):

Directions: Take the word "always." If you had to draw a square of opposition for it, what corner would you put it in? What would be its contrary? What would be its subaltern? (hint: contrast "always" with "never" and "sometimes."

Exercise 4 (Challenge):

Directions: Are the propositions "some unicorns have horns" and "some unicorns do not have horns" a good example of a subcontrary pair? Hint: Aristotle's system only works for objects that exist.

V. Conclusion

As we discussed earlier, understanding the way that the four types of propositions function is crucial to understanding a lot of what we assert about the world around us. Moreover, understanding the relationships among these propositions helps us to see the patterns of arguments and reasoning that we commonly engage in, as well as where fallacies are likely to arise.

VI. Lesson Goals

At the end of this lesson, students will be able to:

- understand what contradictory relations, contrary relations, Universal Affirmative and Universal Negative propositions are;
- map these relations onto the square of opposition;
- transfer knowledge about "all," "some," "no," to "always," "sometimes," "never";
- understand that not all propositions have subalterns, subcontraries, or contraries (but they all have at least one of the three).

Answer Key:

Exercise 1

i. "All dogs are friendly" is a Universal Affirmative proposition. Its contrary would be "no dogs are friendly." Its subaltern would be "some dogs are friendly." Its contradiction would be "some dogs are not friendly." It does not have a subcontrary.

ii. "Some houses are not expensive" is a Particular Negative proposition. Its subcontrary would be "some houses are expensive." It does not have a subaltern. Its contradiction would be "all houses are expensive" (or "no houses are not expensive," which is logically equivalent). It does not have a contrary.

iii. "Some bumblebees are loud" is a Particular Affirmative proposition. Its subcontrary is "Some bumblebees are not loud." Its contradiction is "No bumblebees are loud." It does not have a subaltern. It does not have a contrary.

iv. "No philosophers are musicians" is a Universal Negative proposition. Its contrary is "All philosophers are musicians." Its contradiction is that "Some philosophers are musicians." Its subaltern is "Some philosophers are not musicians." It does not have a subcontrary.

Exercise 2

Answers may vary. Here is one: Some proposition A is the subaltern of another proposition B if and only if the truth of A implies the truth of B, but not the other way around. We know that universal propositions imply the truth of particular propositions, so we know that particular propositions are the subalterns of universal propositions. However, particular propositions do not imply anything in this way. If we say "some dogs are cute," we cannot conclude that "all dogs are cute," nor can we conclude that "some dogs are not cute" or that "no dogs are cute."

Exercise 3

In a square of opposition, the word "always" would belong in the A corner, corresponding to Universal Affirmative propositions. Its contrary would be "never," corresponding to Universal Negative propositions. Its subaltern would be "sometimes," corresponding to Particular Affirmative propositions.

Exercise 4

The propositions "some unicorns have horns" and "some unicorns do not have horns" are not a good example of a subcontrary pair. Given that unicorns do not exist, both propositions are false. However, a subcontrary pair is one in which both propositions *cannot* be false.