Affirming the Consequent and Denying the Antecedent

I. Lesson

In the prior lesson, we introduced the rules of inference for conditional statements known as *Modus Ponens* and *Modus Tollens*. In today's lesson, we'll be looking back at these rules of inference to see what happens when the wrong logical steps are taken during the use of these rules.

There are two major mistakes that can occur in the use of the rules listed above; each of these mistakes corresponds to one of the rules. It is also worth mentioning that both mistakes involve the production of an incorrect second premise of each rule of inference.

i. Affirming the Consequent

The first of these two mistakes is called **Affirming the Consequent**; this name should be revealing once it is noted that Modus Ponens is alternatively referred to as Affirming the Antecedent. **Modus Ponens** takes the form:

(1) If p, then q
(2) p
(3) therefore q

or (for only categorical propositions)

(1) All A's are B's.
(2) C is an A / All C's are A's.
(3) Therefore, C is a B / All C's are B's.

This new fallacy, **Affirming the Consequent**, occurs when an argument takes the following form:

(1) If p, then q
(2) q
(3) therefore p

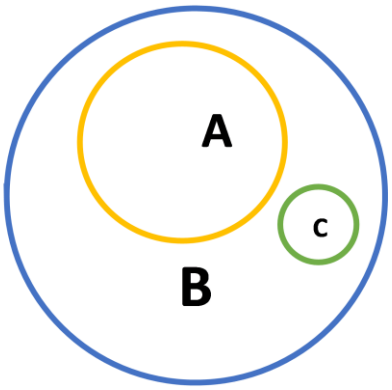
or (for only categorical propositions)

(1) All A's are B's.
(2) C is a B / All C's are B's.
(3) Therefore, C is an A / All C's are A's.

Notice that for the cases with hypothetical propositions, the fallacy switches the second premise and the conclusion of Modus Ponens. This is a mistake, as the conditional only asserts that the

truth of *p* implies that of *q*; it does not assert the opposite, that *q* implies *p*, which the mistaken form needs to be a valid argument.

For the cases with only categorical propositions, the mistake can easily be seen when we realize that there might be some *B*'s that are not *A*'s. Therefore, just because something is a B, that does not mean that that thing is an *A*. This can easily be seen with Euler Circles:



This set of Euler circles says both that (1) all *A*'s are *B*'s and that (2) *C* is a *B*. However, it shows that (1) and (2) could both be the case while *C* is *not* an *A*. This is contradictory with the conclusion, (3). Thus, Affirming the Consequent for only categorical propositions is also *fallacious*.

ii. Denying the Antecedent

The second mistake, **Denying the Antecedent**, relates to Modus Tollens, also known as Denying the Consequent, in a similar way to how Asserting the Consequent relates to Modus Ponens. **Modus Tollens** has the form:

(1) If *p*, then *q*
(2) not *q*
(3) therefore not *p*.

or (for only categorical propositions)

(1) All *A*'s are *B*'s.
(2) *C* is not a *B* / No *C*'s are *B*'s.
(3) Therefore, *C* is not an *A* / No *C*'s are *A*'s.

An argument, making the mistake of **Denying the Antecedent**, has the form:

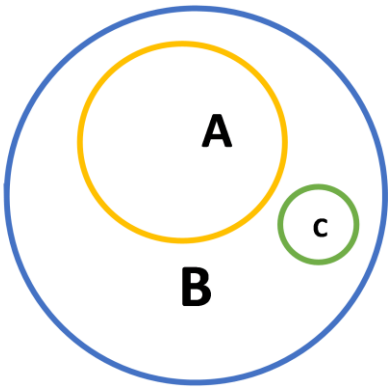
(1) If *p*, then *q*
(2) not *p*
(3) therefore, not *q*

or (for only categorical propositions)

(1) All *A*'s are *B*'s.
(2) *C* is not an *A* / No *C*'s are *A*'s.
(3) Therefore, *C* is not a *B* / No *C*'s are *B*'s.

The reason that this is a mistake is that the conditional links the truth of "*p*" to that of "*q*", such that all instances in which "*p*" is true also have "*q*" as being true. However, there certainly can be cases in which *q* is the case without *p* being the case (examples will make this clearer). It is important to understand these mistakes: an argument which commits one of these mistakes is invalid.

For the cases with only categorical propositions, the mistake can easily be seen when we again realize that there might be some *B*'s that are not *A*'s. Therefore, just because something is not an *A* does not mean that that thing is not a *B*. This can easily be seen with the same Euler Circles:



This set of Euler's circles says both that (1) All *A*'s are *B*'s, and that (2) *C* is not an *A*. However, this is a case where *both* (1) and (2) are the case, while (3) is not the case. That is, *C is* a *B*! Therefore, Denying the Antecedent is *fallacious*.

In using Affirming the Consequent and Denying the Antecedent true premises no longer lead to assuredly true conclusions. Instead, the conclusions reached are either true, but only by happenstance, or they are in fact false. Mistakes in reasoning of this kind are known as *fallacies*. Fallacies can take two forms: there are *formal fallacies*, like the ones shown in this lesson, which have to do with the formal structure of arguments and make these arguments invalid; and there are *informal fallacies*, in which problems in relation to the propositional content of the premises lead to false conclusions. Informal fallacies will be dealt with in later lessons.

II. Examples

In the following section, we will be looking through a couple of examples depicting instances of each of the fallacies listed above:

i. Affirming the Consequent

1. Suppose you know that if a certain dog can smell things up to one mile away, then it will chase after some animal. Now suppose that the dog goes chasing after some animal. From this information, if you were to further conclude that the dog must've been able to smell things up to one mile away, you would be committing the fallacy of Affirming the Consequent. After looking at a concrete example of this fallacy in use, it should be easier to see exactly what makes this fallacious; while the dog did in fact chase after an animal, we are not told whether the dog smelled the animal or whether it used one of its other senses to detect the animal. Because of this, we cannot assuredly conclude that the dog can smell things up to one mile away, as we have no evidence in favor of the truth of this proposition. To see this more clearly, we can formalize the inference as:

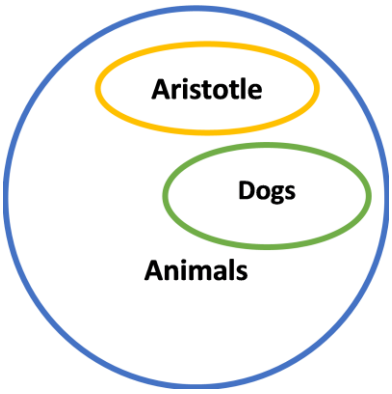
(1) If a certain dog can smell things up to one mile away, then it will chase after some animal.
(2) The dog goes chasing after some animal
(3) Therefore, the dog must've been able to smell things up to one mile away

2. (1) All men are mortal
   (2) Socrates is mortal
   (3) Therefore, Socrates is a man

It is important to note that while the above syllogism has a true conclusion, it is still fallacious. A fallacious type of inference can *sometimes* give true conclusions, but many times it does not. In logic, we care about types of inference that *always* give true conclusions (if the premises are true). Here's an example where Affirming the Consequent fails to give a true conclusion:

3. (1) All dogs are animals
   (2) Aristotle is an animal
   (3) Therefore, Aristotle is a dog.

We can visualize where we went wrong using Euler Circles:



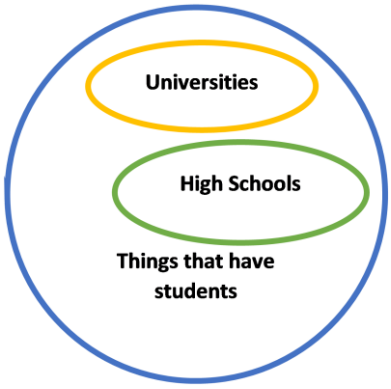
ii. Denying the Antecedent

1. Once again, as in the last lesson, suppose that you now know that if someone falls asleep with a lit candle, their room will catch fire. One day, upon asking your parents about your distant cousin, they tell you that they know nothing about your cousin besides the fact that they have never fallen asleep with a lit candle. If you were to follow the path of Denying the Antecedent, you would reach the conclusion that your cousin's room never caught on fire. This assumption could never be assured by the two premises you received, as there are a multitude of other ways in which a room could catch on fire. Lightning could've stricken the house, or something could have been left on top of the radiator, etc. Given that possibilities of this variety exist, you cannot assume from the premises provided that your cousin's room has not caught on fire. To see this more clearly, we can formalize the inference as:

(1) If someone falls asleep with a lit candle, their room will catch fire
(2) Your cousin has never fallen asleep with a lit candle
(3) Your cousin's room never caught on fire

2. (1) All universities have students
(2) No high schools are universities
(3) Therefore, no high schools have students

We can visualize where we went wrong using Euler Circles:



III. Exercises

Exercise 1

Directions: Answer the following questions

1.) What is the logical form of the fallacy of Affirming the Antecedent?
2.) What is the logical form of the fallacy of Denying the Consequent?
3.) What is a formal fallacy?

Exercise 2

Directions: Determine whether the following chains of reasoning are fallacious or not. If so, fix the premises so as to form valid arguments.

Ex. I know that if it is winter, then it will be snowing in Alaska. One year, I stumbled up to Alaska and realized it was snowing, so I reason that it must be winter.

Answer: This reasoning is fallacious, as it may snow in spring or fall in Alaska. This is an example of Affirming the Consequent. To make this a valid argument the first premise would have to be "if it is snowing in Alaska, it must be winter".

1.) I know that if a caterpillar gets into a cocoon, then it will come out as a butterfly. One day I saw a caterpillar get into its cocoon, so I assume it will soon become a butterfly.
2.) I know that if the Los Angeles Lakers don't win the NBA title, many of the Lakers' fans will cry. At the end of the playoffs, I watch the Lakers win the championship, so I assume the next day that I won't see a single of their fans crying.
3.) I know that if I wear my jade earrings, I'll feel cool. One day, running late for work, I get my small gold hoops instead of my jade earrings, so I assume I won't feel cool for the whole day.

Exercise 3

Directions: Determine whether the following syllogisms are examples of Affirming the Consequent, Denying the Antecedent, or neither. Then determine whether they are examples using hypothetical propositions or only categorical propositions.

1.) (1) All phones are electronic devices.
(2) This computer is not a phone.
(3) Therefore, this computer is not an electronic device.

2.) (1) Either Kant is a philosopher, or Kant is not a philosopher.
(2) Kant is not a philosopher.
(3) Therefore, Kant is not a philosopher.

3.) (1) It's not the case that both Tom is going to the party and Jim is going to the party.
(2) Tom is going to the party.
(3) Therefore, Jim is not going to the party.

4.) (1) If John will go to the store, he will not come back for the rest of the day.
(2) John will not come back for the rest of the day.
(3) Therefore, John will go to the store.

5.) (1) If red is a color, then some people see red.
(2) red is a color.
(3) Therefore, some people see red.

6.) (1) Everything that came through that gate has four wheels.
(2) That cart has four wheels.
(3) Therefore, that cart came through the gate.

7.) (1) If Descartes is a philosopher, then Descartes loves wisdom.
(2) Descartes is not a philosopher.
(3) Therefore, Descartes does not love wisdom.

Exercise 4 (Challenge Question)

Directions: Determine whether the following are examples of Modus Ponens, Modus Tollens, Affirming the Consequent, or Denying the Antecedent. (Hint: First check if the conclusions reached, must be true, due to the information in the premises)

Ex. I know that if my friend does not get a new car, then they won't be driving. Over the next few years, I realize that they haven't driven at all, so I reason that they must've not gotten a new car.

Answer: Affirming the Consequent

1.) I know that if it gets past 2 a.m., I get very tired. One night, after barely being able to keep my eyes open for a few minutes, I decided to check my phone and saw that it was only 1:59 a.m. I reasoned that there must be some medical problem causing me to struggle to keep my eyes open, as I surely couldn't've been tired at such an early hour.

2.) I know that if my cat is coughing, he's spitting up a hairball. For three days, I haven't seen a single hairball, and he is always by my side, so I assume he hasn't been coughing.

3.) I know that if my friends want to buy the new PlayStation, they will have to save up a lot of money first. Months after this realization, they have saved up a good amount of money, so I assume they must've really wanted the new PlayStation.

IV. Conclusion

Hypothetical propositions and categorical propositions are two of the cornerstones of logic and argumentation in general. Last lesson, we learned some ways these statements could be used in arguments to get to true conclusions; this lesson, we went over two ways in which they could be used incorrectly in arguments. It is important that the two fallacies, which we discussed today, are remembered, as these mistakes can easily slip into an argument and invalidate it instantly.

V. Learning Goals:

Following this lesson, the students should be able to identify instances of fallacious arguments following the patterns of *Affirming the Consequent* and *Denying the Antecedent*. The student should also be aware of the relationship between these invalid argumentative forms and Modus ponens and Modus tollens. Finally, students should gain an understanding of what a fallacy is.

Answers Key:

Exercise 1

1. If *p*, then *q*; *q*; therefore *p*
2. If *p*, then *q*; not *p*; therefore, not *q*
3. A fallacy is a mistake in reasoning, which denies the truth of any conclusions reached using said reasoning. (Answers may vary slightly)

Exercise 2

1. This is not fallacious.
2. This is fallacious; to correct this, one would need to change the first premise. The substituted premise will be "if the Los Angeles Lakers win the NBA title, none of the Lakers' fans will cry". This is Denying the Antecedent.
3. This is fallacious; to correct this, one would need to change the first premise. The substituted premise will be "if I don't wear my jade earrings, I won't feel cool". This is Denying the Antecedent.

Exercise 3

1. Denying the Antecedent, only categorical propositions
2. Neither
3. Neither
4. Affirming the Consequent, hypothetical proposition
5. Neither
6. Affirming the Consequent, only categorical propositions
7. Denying the Antecedent, hypothetical proposition

Exercise 3 (Challenge Question)

1. Denying the Antecedent
2. Modus Tollens
3. Affirming the Consequent